The modulated dissipation rate in periodically forced turbulence

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W. Bos, T. Clark, and R. Rubinstein, *Small scale response and modeling of periodically forced turbulence*, PF (2007)

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‘time-dependent’ (nonstationary) or spatially evolving turbulence in general

– recovery following re-attachment in backstep flow (T. Gatski)

– boundary layers with change in wall conditions (rough/smooth, smooth/rough) (Smits and Wood, ARFM (1985); Woodruff and Nwafor (1992))

– ‘ramp flow’ (Rubinstein, Clark, Livescu, Luo, JoT (2004)): transition between self-similar states.

Such problems present major challenges for modeling.

classic problem: pipe flow with oscillating pressure gradient

$$\nabla P = \nabla \bar{P} + \nabla \tilde{P} \cos(\omega t) \qquad \nabla \tilde{P} \ll \nabla \bar{P}$$
$$\tau_w = \bar{\tau}_w + \tilde{\tau}_w \cos(\omega t + \phi_\tau)$$
$$U_c = \bar{U}_c + \tilde{U}_c \cos(\omega t + \phi_U)$$

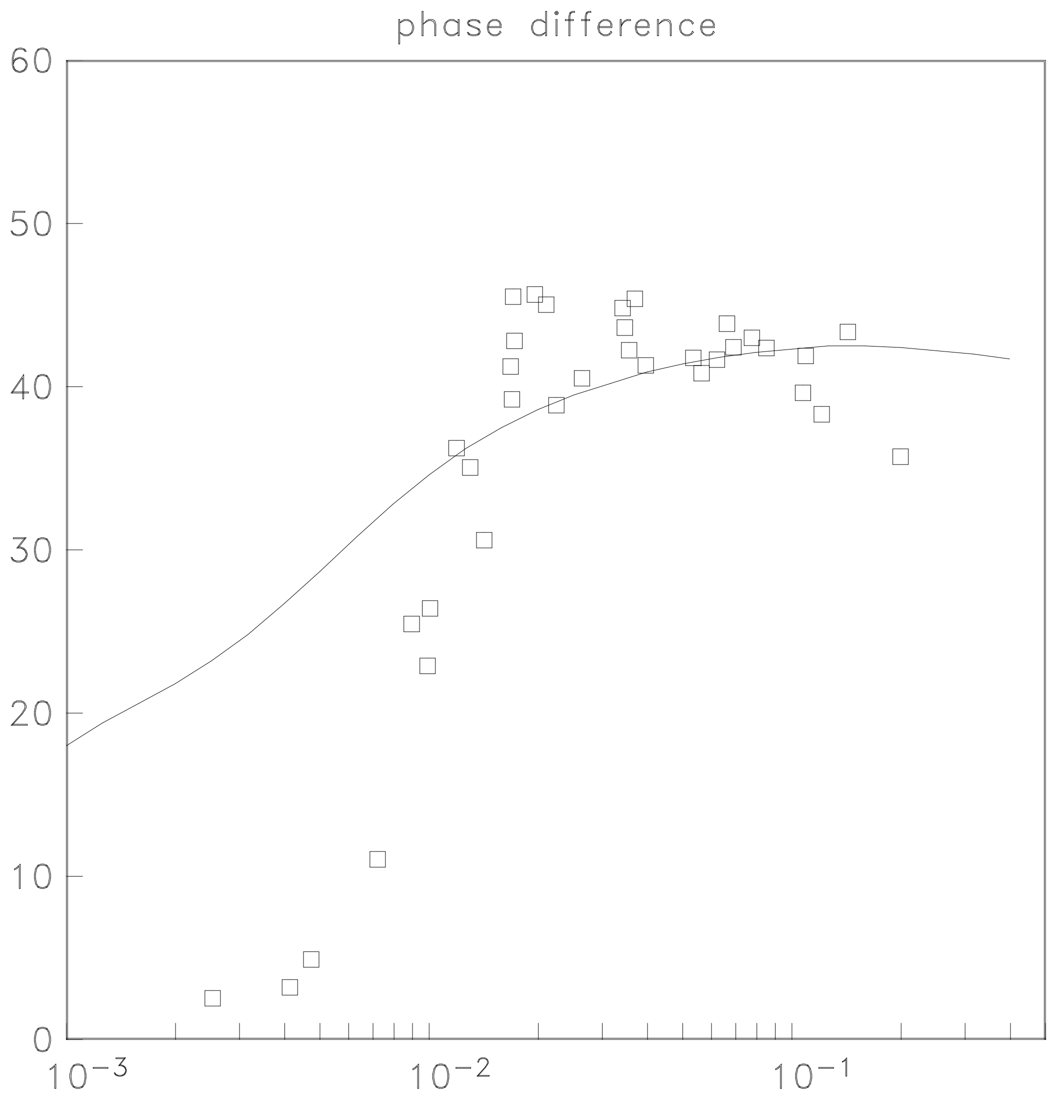
– Obvious limits: **static** $\omega \approx 0$, **frozen** $\omega \to \infty$.

(same limits for laminar flow)

– Measured values of amplitude ratio $\tilde{\tau}/\tilde{u}_c$ and phase shift $\phi_\tau - \phi_U$ (V. Yakhot) were compared to results of linearizing a two-equation model about the steady solution.

– Is the ‘static’ model $\tau = C_\nu \frac{k^2}{\epsilon}\frac{\partial U}{\partial y}$ adequate, or is the time-dependent model $\dot{\tau} = -C_S \frac{\epsilon}{k}\tau + C_R \frac{\partial U}{\partial y} k$ preferable?

– Conclusion: there is no significant difference. Model predictions for $\tilde{\tau}/\tilde{u}_c$ were good, but $\phi_\tau - \phi_U$ showed **much too gradual** transition between static and frozen limits, with either model.


phase difference
60
50
40
30
20
10
0
10^{-3}
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periodically forced turbulence

* problem introduced by Lohse – possibility of *resonant response* in kinetic energy near *critical frequency* $\bar{\omega} \propto \bar{\epsilon}/\bar{k}$

* closure, shell model, and DNS studies (von der Heydt *et al*, Kuczaj *et al*)

* measurements using periodic grid (van de Water)

problem formulation

Homogeneous isotropic turbulence is forced periodically by
$P(\kappa,t) = \bar{P}(\kappa) + \tilde{P}(\kappa)\,\mathrm{cos}(\omega t)$ where $\tilde{P}(\kappa) = \varepsilon \bar{P}(\kappa)$ and $\varepsilon \ll 1$

$E(\kappa,t) = \bar{E}(\kappa) + \tilde{E}(\kappa)\,\mathrm{cos}(\omega t + \phi_E(\kappa))$

$\tilde{F}(\kappa) = \tilde{E}(\kappa)\,\mathrm{cos}\,\phi_E(\kappa) \quad \tilde{G}(\kappa) = \tilde{E}(\kappa)\,\mathrm{sin}\,\phi_E(\kappa)$

$k(t) = \bar{k} + \tilde{k}\,\mathrm{cos}(\omega t + \phi_k) \quad \epsilon(t) = \bar{\epsilon} + \tilde{\epsilon}\,\mathrm{cos}(\omega t + \phi_\epsilon)$

Oscillations characterized by *phase averages* $\tilde{k}, \tilde{\epsilon}$ and *phase shifts* ϕ_k, ϕ_ϵ.

General spectral evolution equation for closure

$$\dot{E}(\kappa,t) = P(\kappa,t) - \frac{\partial}{\partial\kappa}\mathcal{F}[E(\kappa,t)] - 2\nu\kappa^2 E(\kappa,t)$$

Spectral time averages satisfy the static balance

$$0 = \bar{P}(\kappa) - \frac{\partial}{\partial\kappa}\mathcal{F}[\bar{E}(\kappa)] - 2\nu\kappa^2 \bar{E}(\kappa)$$

and spectral phase averages satisfy

$$\begin{aligned} -\omega\tilde{G}(\kappa) &= \tilde{P}(\kappa) - \mathcal{L}[\tilde{F}(\kappa)] - 2\nu\kappa^2\tilde{F}(\kappa) \\ -\omega\tilde{F}(\kappa) &= \mathcal{L}[\tilde{G}(\kappa)] + 2\nu\kappa^2\tilde{G}(\kappa) \end{aligned}$$

where $\mathcal{L}$ is the energy transfer linearized about $\bar{E}$:

$$\mathcal{L}[\Phi(\kappa)] = \frac{\partial}{\partial\kappa}\left(\frac{\delta\mathcal{F}}{\delta E}\right)_{\bar{E}}[\Phi(\kappa)]$$

two obvious limits:

1. **static limit** $\omega \to 0$: then $\phi_k \approx \phi_\epsilon \approx 0$.

2. **frozen limit** $\omega \to \infty$: then ('rapid distortion theory')

$$\tilde{k} \approx \frac{1}{\omega}\int_0^\infty d\kappa\ \tilde{P}(\kappa) \approx \tilde{P}/\omega \qquad \tilde{\epsilon} \approx \omega^{-1}\int_0^\infty d\kappa\ 2\nu\kappa^2\tilde{P}(\kappa) \approx \nu\kappa_P^2\tilde{P}/\omega$$

and $\phi_k \approx \pi/2(\approx \phi_\epsilon?)$

The static and frozen limits show no oscillatory dynamics.

Filtering effect: at asymptotically large ω, oscillations are *not transfered to small scales*; confined to the production scales.

Interesting properties can exist at **intermediate** frequencies.

closure studies

1. Simplified closure model studies of ϕ_k.

2. Periodically forced turbulence simulated over a range of frequencies and **Reynolds numbers** using EDQNM (PF 2007).

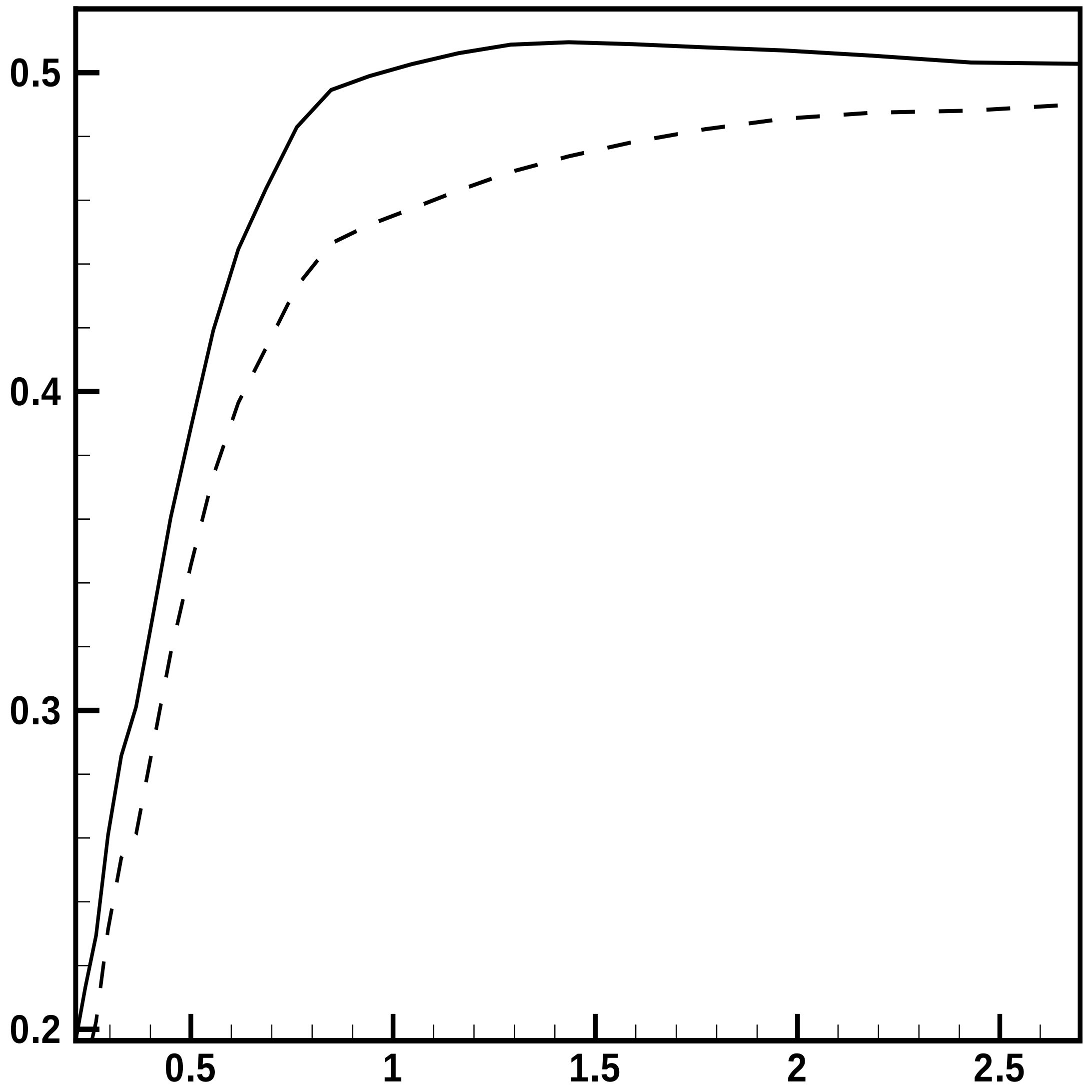
closure studies 1

– Rapid transition of ϕ_k compared to models (two-equation, 'LES'):

* We did 'deterministic LES' (replace the small scales in closure with subgrid model based on arguments leading to Smagorinsky)

* 50/500 modes are resolved explicitly; they contain 80% of the total steady-state energy.

* The phase shift between static and frozen limits is **too gradual**: large eddy simulation = slow eddy simulation.



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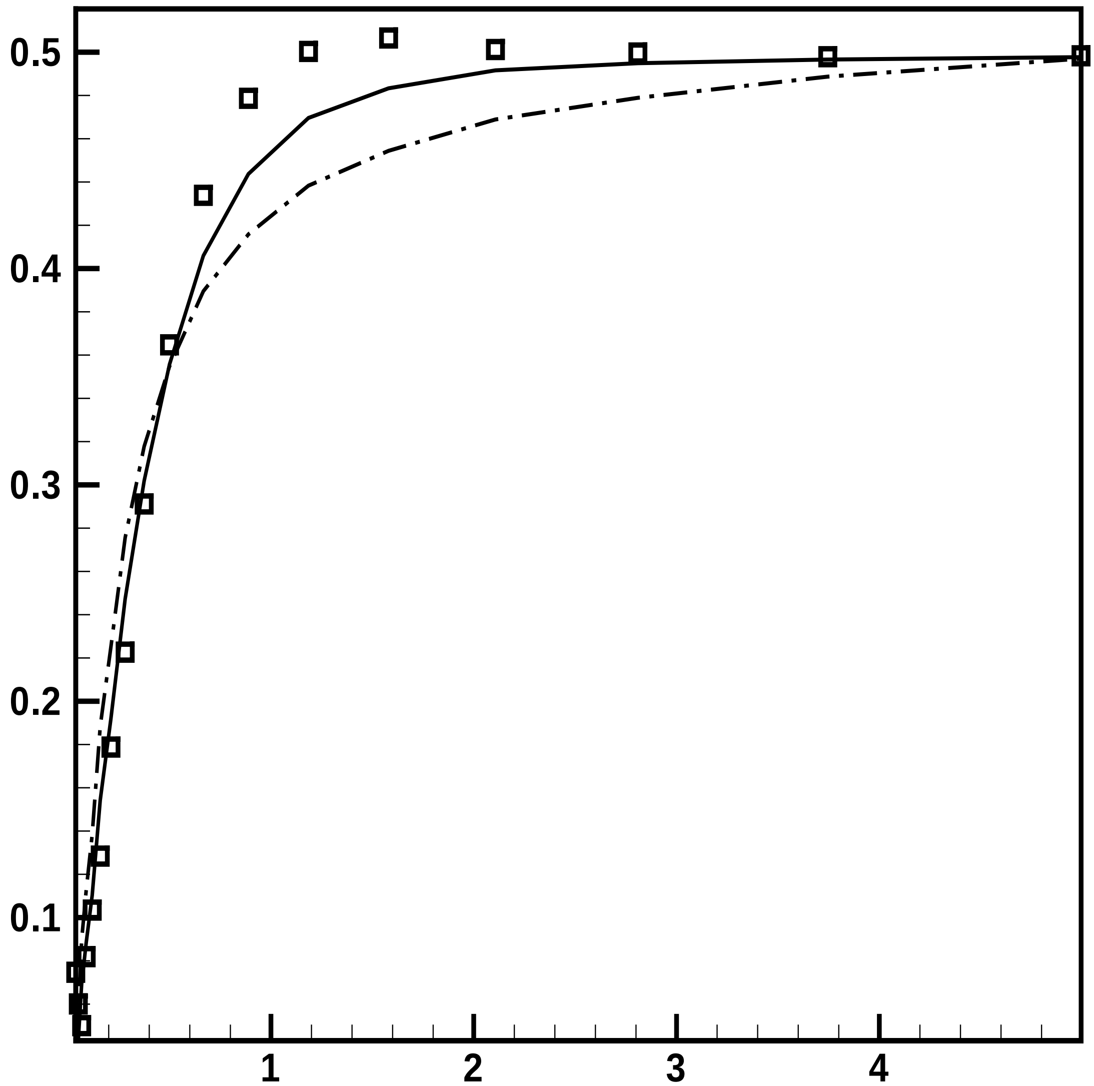
2.5


0.5
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1
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3
4

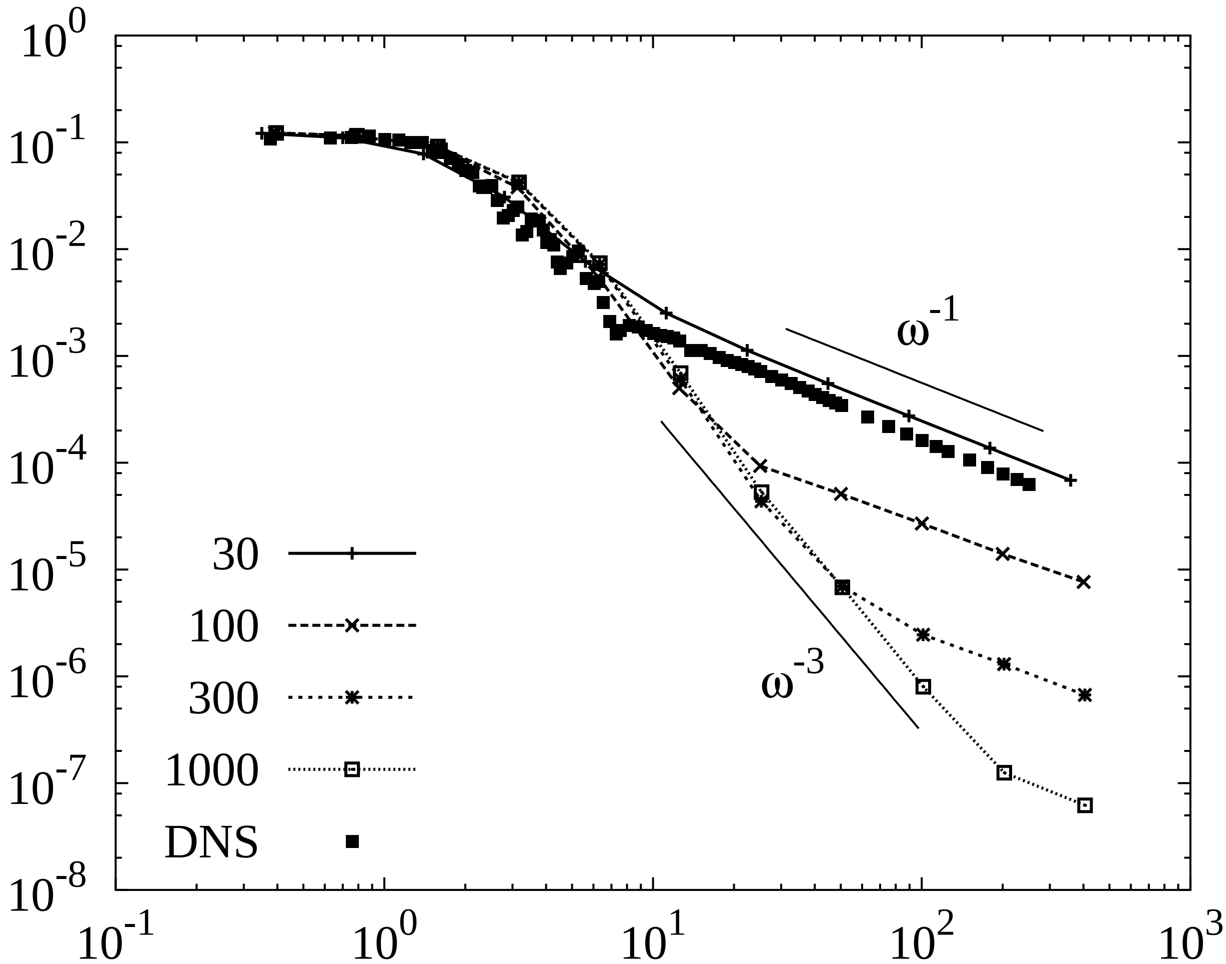
closure studies 2: modulated dissipation rate

* $\tilde{\epsilon}$ shows two limits:

– at fixed Re with $\omega \to \infty$, $\tilde{\epsilon} \sim \nu\omega^{-1}$.

– at fixed ω as $Re \to \infty$, $\tilde{\epsilon} \sim \omega^{-3}$.

* ϕ_ϵ shows complex dependence on both ω and Re: peak in ϕ_ϵ scales roughly as $Re^{1/2}$.



10^{0}
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10^{-8}
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ω^{-3}
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100
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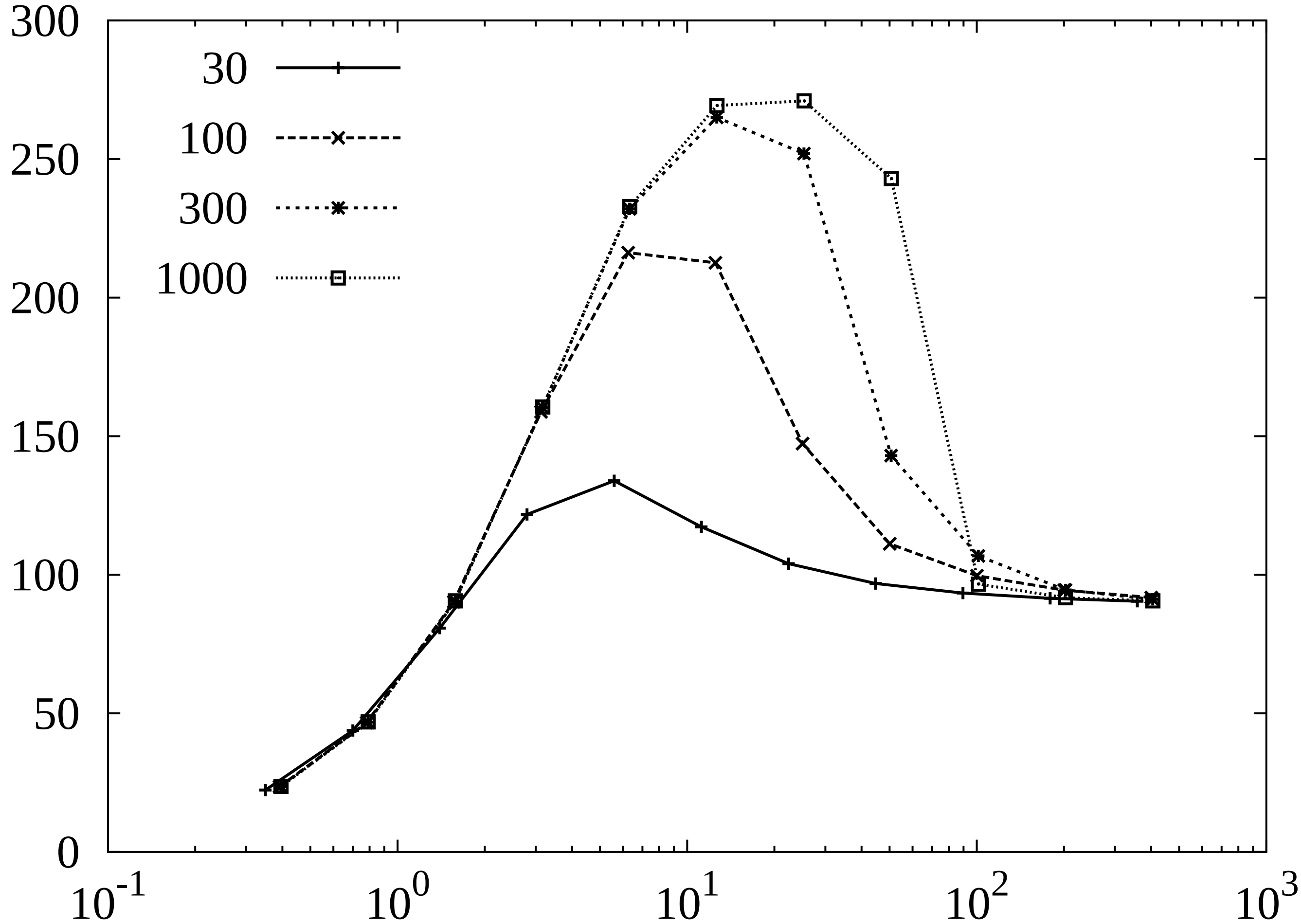
1000
DNS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10^{2}
10^{3}

corrections to the leading order solution: high ω

$$\tilde{E}(\kappa)\cos(\phi_E(\kappa)) \approx 0 \quad \tilde{E}(\kappa)\sin(\phi_E(\kappa)) \approx -\frac{1}{\omega}\tilde{P}(\kappa).$$

'Neumann series' in powers of $\mathcal{L}\omega^{-1}$; but this series is divergent.

$$\begin{aligned}\tilde{E}(\kappa)\cos(\phi_E(\kappa)) &= \omega^{-2}\mathcal{L}[\tilde{P}(\kappa)] \\ \tilde{E}(\kappa)\sin(\phi_E(\kappa)) &= -\omega^{-1}\tilde{P}(\kappa)+\omega^{-3}\mathcal{L}^2[\tilde{P}(\kappa)]\end{aligned}$$

The leading order does not depend on nonlinearity. To understand nonlinear effects, we will analyze a simple model of energy transfer.

CMSB generalized Heisenberg closure

$$\mathcal{F}[E(\kappa)] = \int_0^\kappa d\mu\ \mu^2 E(\mu) \int_\kappa^\infty dp\ E(p)\theta(p) - \int_0^\kappa d\mu\ \mu^4 \int_\kappa^\infty dp\ \frac{E(p)^2\theta(p)}{p^2}$$

where $\theta(\kappa) = [\kappa^3 E(\kappa)]^{-1/2}$. (Rubinstein and Clark, TCFD: compare to Canuto-Dubovikov model)

In the linearized transfer for this model, $\mathcal{L}[\tilde{P}] \propto \tilde{P}$ always holds, except for the term $\mathcal{L}_{NL}[\tilde{P}(\kappa)] = \sqrt{\frac{\bar{E}(\kappa)}{\kappa^3}} \int_0^\kappa d\mu\ \mu^2 \tilde{P}(\mu)$. This term **permits the oscillations to extend to all scales of motion**: its contribution is subdominant in ω.

We find

$$\tilde{\epsilon} \sim \tilde{P}\kappa_P^2 \begin{cases} \omega^{-1}\nu & Re \text{ fixed, } \omega \to \infty \\ \omega^{-3}\bar{\epsilon} & \omega \text{ fixed, } Re \to \infty \end{cases}$$

in agreement with computations.

Note elegant scaling argument based on properties of distant interactions (W. Bos).

conclusions

– Periodically forced turbulence provides an interesting viewpoint on (statistically) unsteady properties. Unsteadiness is 'frozen' in time.

* Too slow phase growth of ϕ_k : comparable to pipe flow?

* Distant interactions cause $\tilde{\epsilon} \sim \omega^{-3}$ and

* strong Reynolds number dependence of ϕ_ϵ.

– Reynolds number dependence of small scale dynamics in turbulent transients may have implications for LES.