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Intermittency in the Miscible Rayleigh – Taylor Turbulence

Antonio Celani[†] Andrea Mazzino[‡§] Lara Vozella[‡§]

[†]Centre National de la Recherche Scientifique, Institut non Linéaire de Nice (France)

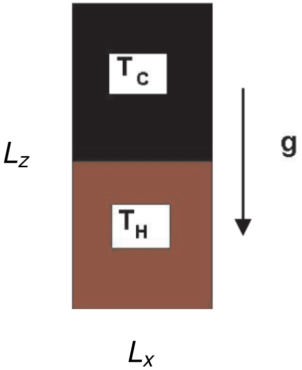
[‡]Università di Genova, Dipartimento di Fisica (Italy)

[§]Istituto Nazionale di Fisica Nucleare, sezione di Genova (Italy)

Objective

Numerical Investigation
of statistical properties of flow
in the mixing zone

The framework



- miscible, homogeneuos fluids (temperature $T_H > T_C$) in gravitational field $\boldsymbol{g}$
- two-dimensional case:
  - theoretical results are more controversial
  - a larger numerical resolution

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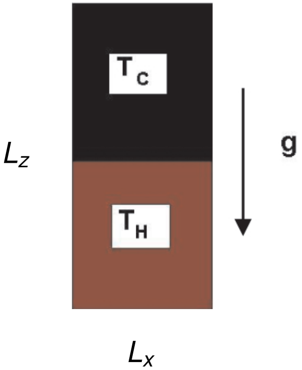

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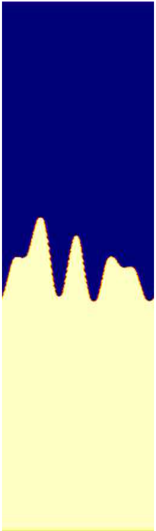
Time evolution: a qualitative idea



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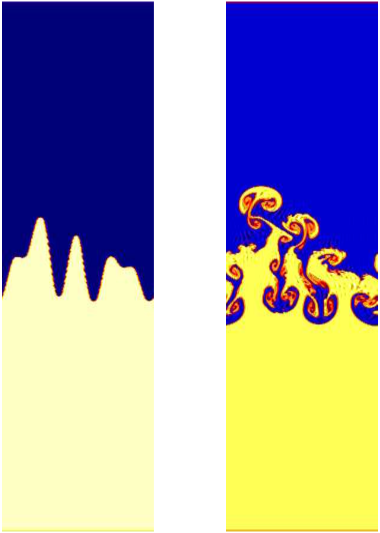

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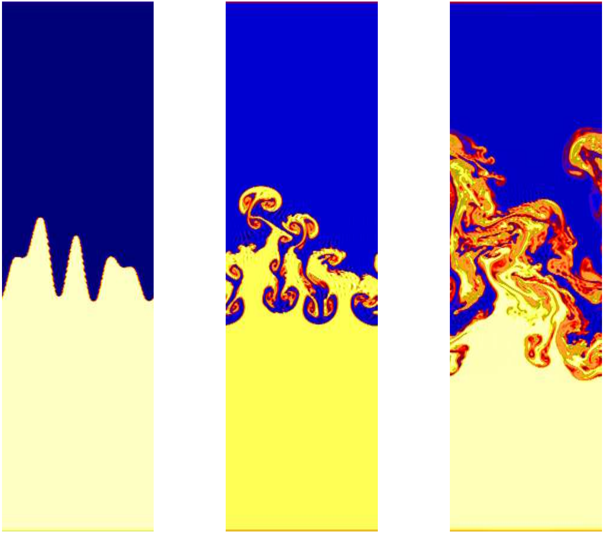

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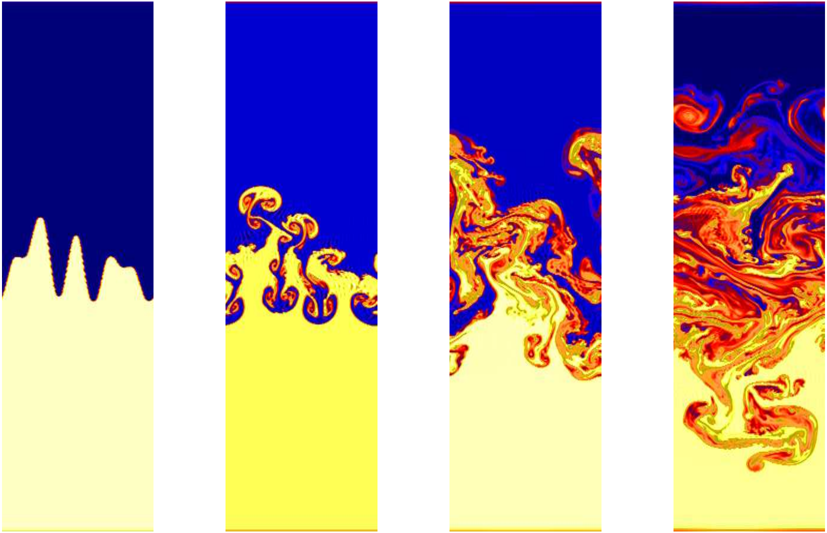
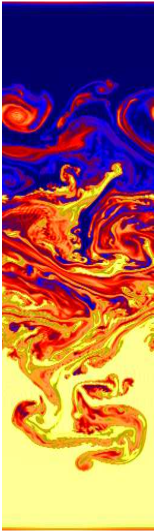


Turbulent stage: system observables



system observables

- mixing layer width
- field increments at different scales r
- global observables (e.g. Nu *vs* Ra, Re *vs* Ra)

Phenomenology of **2D** Rayleigh-Taylor turbulence [(M. Chertkov Phys. Rev. Lett. 91, (2003)]

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\begin{aligned}
\partial_t \mathbf{v} + \mathbf{v} \cdot \boldsymbol{\partial} \mathbf{v} &= -\frac{\partial p}{\varrho_o} + \nu \partial^2 \mathbf{v} - \beta \mathbf{g}(T - T_o) \\
\boldsymbol{\partial} \cdot \mathbf{v} &= 0 \\
\partial_t T + \mathbf{v} \cdot \partial T &= D \partial^2 T
\end{aligned}
$$

mean field predictions

- mixing layer width

$$L(t) \sim \beta g \Theta t^2$$

- field increments scaling (at scales $r \ll L$)

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\begin{aligned}
S_n^{\mathbf{v}}(\mathbf{r}, t) &= (\delta_r v)^n \sim (\beta g)^{-1/5} \Theta^{4/5} r^{3n/5} t^{-n/5} \\
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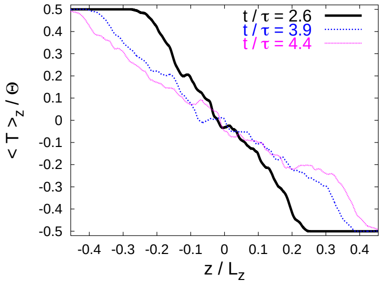
Mean profile of temperature



- almost linear behaviour in the mixing zone

$\Rightarrow$ clue of relation between 2D Rayleigh-Taylor turbulence and 2D Boussinesq turbulent convection⋆

⋆ A. Celani, A. Mazzino and M. Vergassola, Phys. Fluids 13, (2001);
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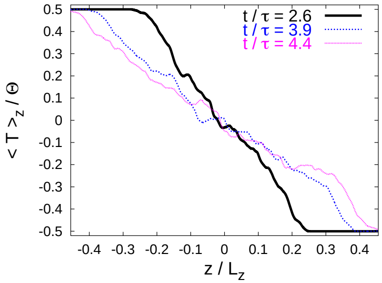

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Spatial scaling laws of fields increments

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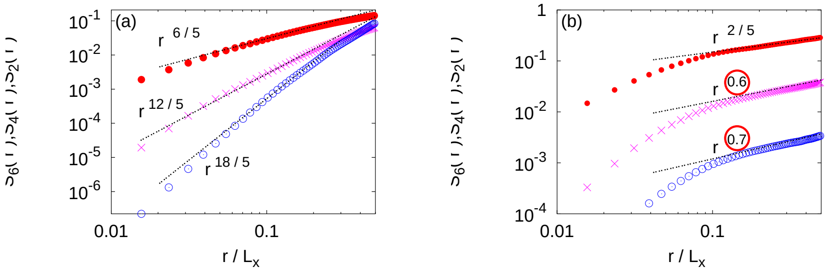

- temperature field: intermittency corrections ⇒ non-dimensional scaling exponents by [A. Celani, A. Mazzino and M. Vergassola Phys. Fluids 13, (2001)]

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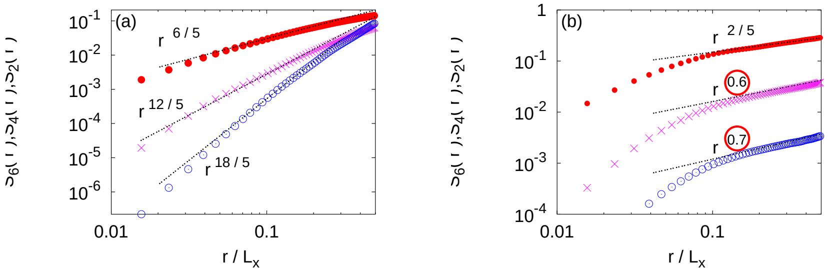

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Ultimate state of thermal convection

- control parameter: Rayleigh number (mean temperature gradient)
- system answers: Reynolds number (root-mean-square velocity), Nusselt number (turbulent heat flux)

- Observation of “Ultimate state of thermal convection”: D. Lohse and F. Toschi, Phys. Rev. Lett. 90, 034502 (2003)

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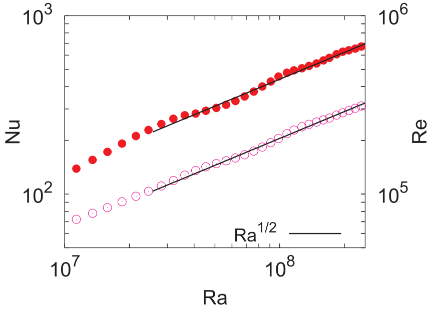

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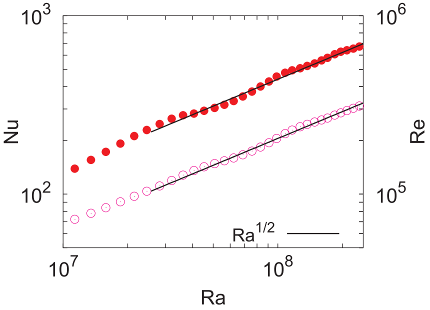

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Conclusions

2D Rayleigh–Taylor turbulence:

- **Statistics of velocity field**
  - close-to-Gaussian
  - Bolgiano–Obukhov scaling
- **Statistics of temperature field**
  - low order: Bolgiano–Obukhov scaling
  - highter order: intermittency corrections ⇒ RT turbulence corresponds to the case driven by a linear profile which gradient decreases adiabatically in time
- **Global statistical observables**:
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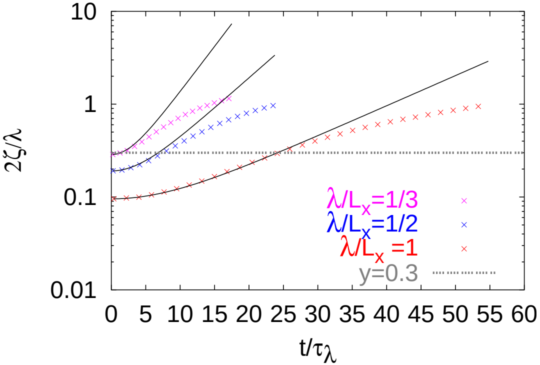
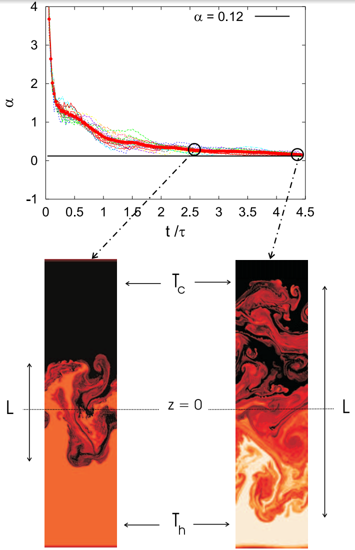
outline

The linear phase



$$\zeta(t) = \zeta(t_0) \cosh\left(\sqrt{\frac{2\pi}{\lambda} g \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}}\, t\right)$$

The mixing layer width



- mixing layer $L(t) = \alpha \mathcal{A} g t^2$
  $\Longrightarrow \alpha = [\frac{1}{\mathcal{A}g}] \frac{dL}{dt^2}$

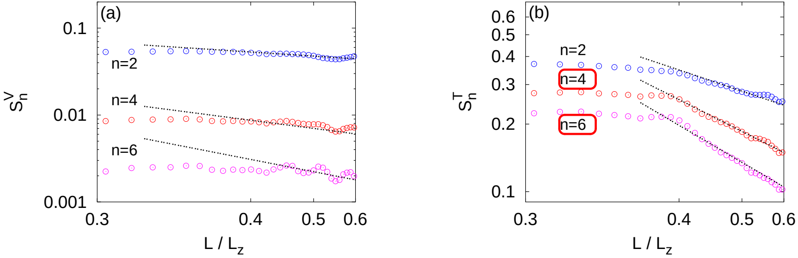
- $\mathcal{A} = \frac{\varrho_2 - \varrho_1}{\varrho_2 + \varrho_1} = \frac{\beta\Theta}{2}$
  Atwood number

- $\tau = \left[\frac{L_z}{\mathcal{A}g}\right]^{1/2}$ characteristic time scale

$\Rightarrow$ T. T. Clark, Phys. Fluids 15, (2003)

The temporal scaling of fields increments

dimensional predictions : $S_n^v(\mathbf{r},t) \sim t^{\frac{-n}{5}} \sim L^{\frac{-2n}{5}}$ $\qquad S_n^T(\mathbf{r},t) \sim t^{\frac{-2n}{5}} \sim L^{\frac{-4n}{5}}$



- temperature field displays temporal intermittency (beginning from $n > 2$)

$$S_n^T(r) \sim \Theta^n [r/L]^{n/5} [r/L]^{-\sigma_n}$$

by [Celani&al. Phys. Fluids (2001)] $\Rightarrow$ $S_4^T \sim L^{-0.6}$ and $S_6^T \sim L^{-0.7}$

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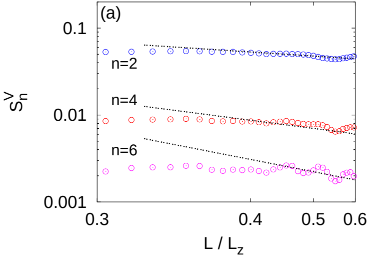

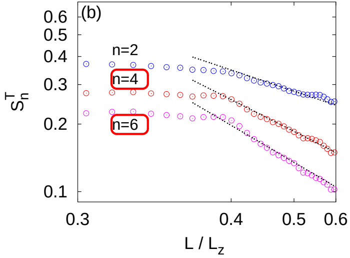

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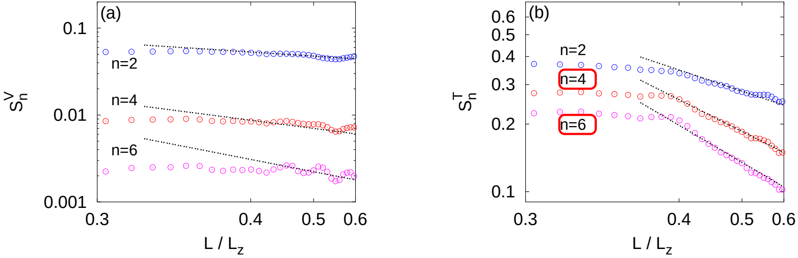

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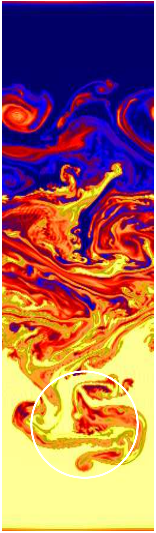
Non dimensional scaling exponents and fronts



- **Fronts**: abrupt changes in the spatial structure of the temperature field

outline of appendix

4 Other numerical results
- The linear phase
- The turbulent phase