Comparison of Tensor Representation of Velocity-Pressure Gradient, Pressure-Strain and Pressure-Velocity Correlations with Plane Channel Flow DNS data

11.ETC, june 25-28 2007, Porto [PRT]

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June 26, 2007

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- Tensor Representation for ϕ_{ij} coupled with ε_{ij}
- Tensor Representations for ϕ_{ij} present in the Literature

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Purpose

Why do we need to analyse the Tensor Representation of pressure-velocity correlations ?

• Turbulent Correlations containing pressure Fluctuation p'

$\Rightarrow$ All these terms are present in RSM closures

We will study these terms to get a best prediction of the wall-bounded turbulent flows

The Redistribution term : $\phi_{ij} = \overline{p'\left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} - \frac{2}{3}\frac{\partial u'_k}{\partial x_k}\delta_{ij}\right)}$

The Pressure-diffusion term : $d^{(p)}_{ij} = -\frac{\partial}{\partial x_\ell}[\overline{p'u''_j}\delta_{i\ell} + \overline{p'u''_i}\delta_{j\ell}]$

$\Longrightarrow$ The terms ϕ_{ij} and $d^{(p)}_{ij}$ can be recombined into a single term :

$$\Pi_{ij} = \overline{u'_i\frac{\partial p'}{\partial x_j}} + \overline{u'_j\frac{\partial p'}{\partial x_i}} = \phi_{ij} + d^{(p)}_{ij}$$

with Π_{ij} the Velocity-Pressure gradient tensor

• ϕ_{ij} and $d^{(p)}_{ij}$ are both predominant terms in RSM closures
$\Longrightarrow$ The redistribution term ϕ_{ij} is not properly modelled

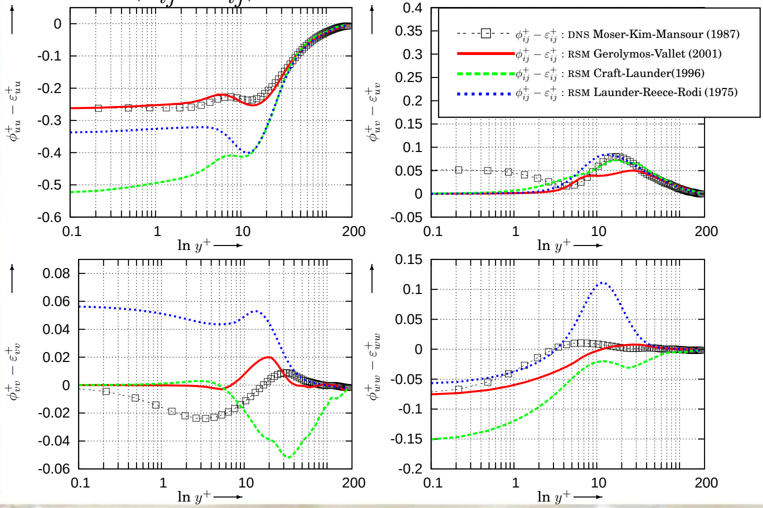
First part

The existing Tensor Representation analysis for Redistribution term ϕ_{ij}

With incompressible DNS results of Kim et al.(1987) for a Plane Channel Flow $(Re_{\tau_w} = 180;\ M_{\mathrm{B}} \equiv 0)$

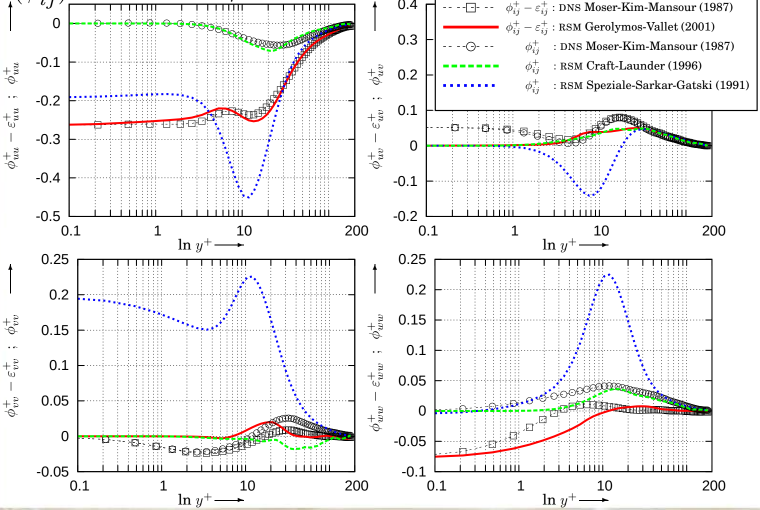
- Tensor Representations for ϕ_{ij} coupled with ε_{ij}

$(\phi^{+}_{ij} - \varepsilon^{+}_{ij})$ models at $Re_\tau = 180$



- **Tensor Representations for ϕ_{ij}**

(ϕ^+_{ij}) models at $Re_\tau = 180$



Second part

Splitting method and resolution with Green's Functions

- Poisson Equation

$$\nabla^2 p' = \underbrace{\frac{\partial^2 \tau_{ij'}}{\partial x_i \partial x_j}}_{Q_\tau} + \underbrace{\left[-\bar{\rho}\left(\frac{\partial u'_i}{\partial x_j}\frac{\partial u'_j}{\partial x_i} - \overline{\frac{\partial u'_i}{\partial x_j}\frac{\partial u'_j}{\partial x_i}}\right)\right]}_{Q_s} + \underbrace{\left[-2\bar{\rho}\left(\frac{\partial u'_i}{\partial x_j}\frac{\partial \bar{u}_j}{\partial x_i}\right)\right]}_{Q_r}$$

and boundary conditions at a fixed solid wall:

$$\frac{\partial p}{\partial n} = \frac{\partial \tau_{nj}}{\partial x_j} + \rho f_{\mathrm{v}_n} \quad : \quad \forall \vec{x} \in \partial\mathfrak{V}_w$$

with the splitting method :

$$\nabla^2 p'_\tau = Q_\tau \quad ; \quad \frac{\partial p'_\tau}{\partial n} = \frac{\partial \tau'_{nj}}{\partial x_j} \quad : \quad \forall \vec{x} \in \partial\mathfrak{V}_w$$
$$\nabla^2 p'_s = Q_s \quad ; \quad \frac{\partial p'_s}{\partial n} = 0 \quad : \quad \forall \vec{x} \in \partial\mathfrak{V}_w$$
$$\nabla^2 p'_r = Q_r \quad ; \quad \frac{\partial p'_r}{\partial n} = 0 \quad : \quad \forall \vec{x} \in \partial\mathfrak{V}_w$$

• Green's Functions

the originality of this splitting method is to use the two following Green's Functions

•Green's function with walls according to Kim (1989):

$$\hat{p}'_{(m)}(\kappa_x, y, \kappa_z, t) = \left\{ \int_{-\frac{1}{2}L_y}^{+\frac{1}{2}L_y} \left[G_{\mathrm{Kim}}(\kappa, y, Y)\; \hat{Q}'_{(m)}(\kappa_x, y, \kappa_z, t) \right] dY \right\} + \hat{p}'_{(m)_{\mathrm{BCs}}}(\kappa_x, y, \kappa_z, t)$$

•the free-space Green's function

$$\hat{p}'_{(m;\mathfrak{B})}(\kappa_x, y, \kappa_z, t) = \left\{ \int_{-\frac{1}{2}L_y}^{+\frac{1}{2}L_y} \left[G_{\infty}(\kappa, y, Y)\; \hat{Q}'_{(m)}(\kappa_x, y, \kappa_z, t) \right] dY \right\}$$

$$\Longrightarrow \hat{p}'_{(m;w)}(\kappa_x, y, \kappa_z, t) = \hat{p}'_{(m)}(\kappa_x, y, \kappa_z, t) - \hat{p}'_{(m;\mathfrak{B})}$$

where:

$$\left[\frac{d^2}{dy^2} - \kappa^2\right] G_{\mathrm{Kim}}(\kappa, y, Y) = \delta(y - Y) \quad ; \quad \left[\frac{dG_{\mathrm{Kim}}}{dy}\right](\kappa, y = \pm\tfrac{1}{2}L_y, Y) = 0$$

$$\left[\frac{d^2}{dy^2} - \kappa^2\right] \hat{p}'_{(m)_{\mathrm{BCs}}}(\kappa_x, y, \kappa_z, t) = 0 \quad ; \quad \left[\frac{d}{dy}\hat{p}'_{(m)_{\mathrm{BCs}}}\right](\kappa_x, y = \pm\tfrac{1}{2}L_y, \kappa_z, t) = \hat{B}'_{(m)_{\pm}}(\kappa_x, \kappa_z, t)$$

$$\left[\frac{d^2}{dy^2} - \kappa^2\right] G_{\infty}(\kappa, y, Y) = \delta(y - Y) \quad ; \quad \lim_{|y-Y| \longrightarrow \infty} G_{\infty}(\kappa, y, Y) = 0$$

Third part

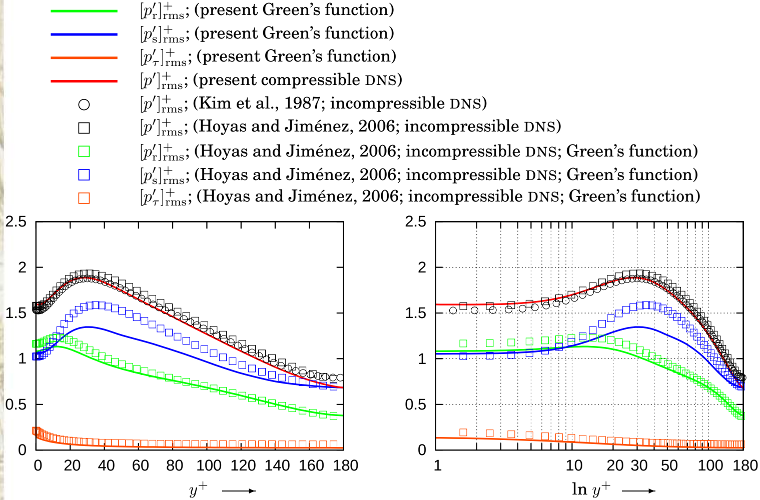
Results of the Splitting Method

With compressible DNS results for a Plane Channel Flow
$(Re_{\tau_w} = 185;\ M_{\mathrm{B}_w} = 0.3;\ \bar{M}_{\mathrm{CL}} = 0.35)$

G.A. Gerolymos, D.Sénéchal and I. Vallet, **AIAA Paper 2007-3863**
37th AIAA Fluid Dyn. Conference and Exhibit, 25-28 june 2007, Miami [FL]

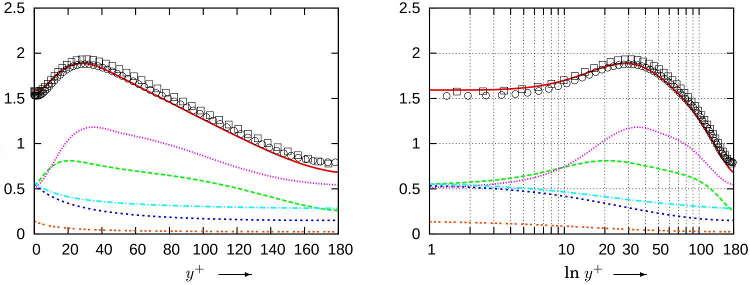
G.A. Gerolymos, D.Sénéchal and I. Vallet, **AIAA Paper 2007-4196**
18th AIAA Comput. Fluid Dyn. Conference, 25-28 june 2007, Miami [FL]

• rms-values p', $p'_{(s)}$, $p'_{(r)}$ and $p'_{(\tau)}$

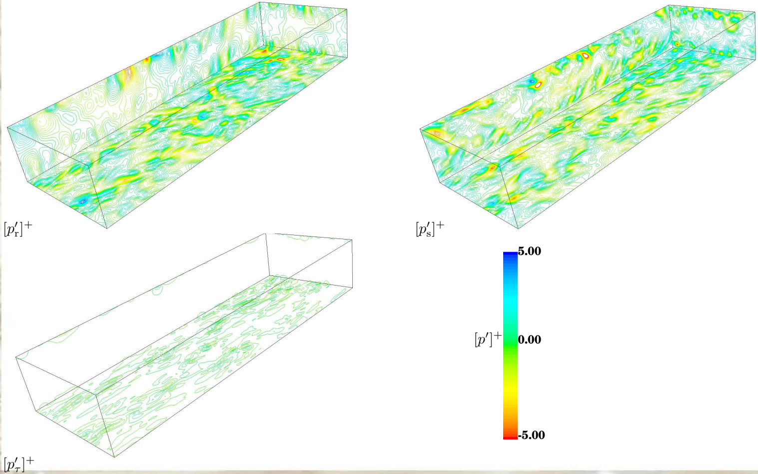


• rms-values p', $p'_{(s)}$, $p'_{(r)}$, $p'_{(\tau)}$ and $(+)$ $[p'_{(s,w)}\ ;\ p'_{(r,w)}]$

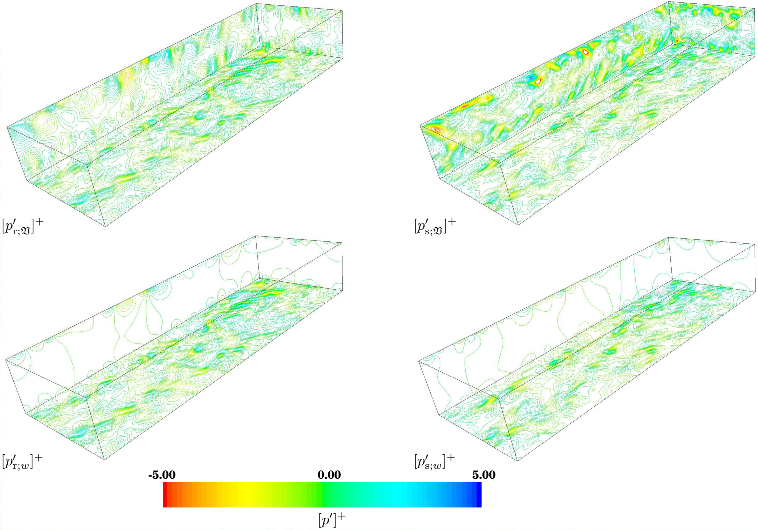
- $[p'_{(\mathrm{r};\mathfrak{V})}]^+_{\mathrm{rms}}$; (present Green's function)
- $[p'_{(\mathrm{r};w)}]^+_{\mathrm{rms}}$; (present Green's function)
- $[p'_{(\mathrm{s};\mathfrak{V})}]^+_{\mathrm{rms}}$; (present Green's function)
- $[p'_{(\mathrm{s};w)}]^+_{\mathrm{rms}}$; (present Green's function)
- $[p'_{\tau}]^+_{\mathrm{rms}}$; (present Green's function)
- $[p']^+_{\mathrm{rms}}$; (present compressible DNS)
- ○ $[p']^+_{\mathrm{rms}}$; (Kim et al., 1987; incompressible DNS)
- □ $[p']^+_{\mathrm{rms}}$; (Hoyas and Jiménez, 2006; incompressible DNS)



- rms-values : $p'_{(s)}$, $p'_{(r)}$ and $p'_{(\tau)}$



- rms-values : $[p'_{\mathrm{s};\mathfrak{V}}]^+$, $[p'_{\mathrm{r};\mathfrak{V}}]^+$, $[p'_{\mathrm{s};w}]^+$ and $[p'_{\mathrm{r};w}]^+$

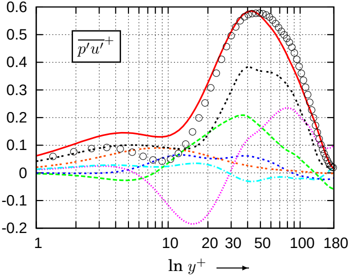


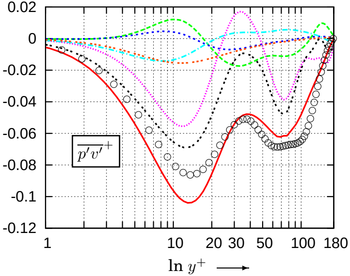
Fourth part

the Turbulent Correlations Splitting

- $\overline{p'u'_i}^{+}$, $\overline{p'_{\mathrm{s};\mathfrak{V}}u'_i}^{+}$, $\overline{p'_{\mathrm{r};\mathfrak{V}}u'_i}^{+}$, $\overline{p'_{\mathrm{s};w}u'_i}^{+}$, $\overline{p'_{\mathrm{r};w}u'_i}^{+}$ and $\overline{p'_{(\tau)}u'_i}$

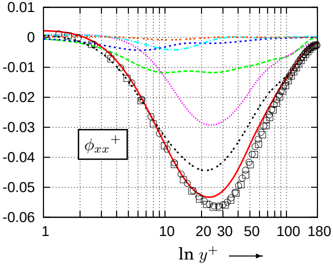
$[\overline{p'_{(\mathrm{r};\mathfrak{V})}u'_i}]^{+}$; (present Green's function)
$[\overline{p'_{(\mathrm{r};w)}u'_i}]^{+}$; (present Green's function)
$[\overline{p'_{(\mathrm{s};\mathfrak{V})}u'_i}]^{+}$; (present Green's function)
$[\overline{p'_{(\mathrm{s};w)}u'_i}]^{+}$; (present Green's function)
$[\overline{p'_{\tau}u'_i}]^{+}$; (present Green's function)
$[\overline{p'_{(\mathrm{r};\mathfrak{V})}u'_i}]^{+} + [\overline{p'_{(\mathrm{r};w)}u'_i}]^{+} + [\overline{p'_{(\mathrm{s};\mathfrak{V})}u'_i}]^{+} + [\overline{p'_{(\mathrm{s};w)}u'_i}]^{+} + [\overline{p'_{\tau}u'_i}]^{+}$; (present compressible DNS)
$[\overline{p'u'_i}]^{+}$; (present compressible DNS)
$[\overline{p'u'_i}]^{+}$; (Kim et al., 1987; incompressible DNS)

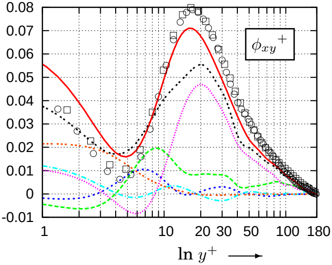




- $\phi^{+}_{ij}, \phi^{(s;\mathfrak{V})+}_{ij}, \phi^{(r;\mathfrak{V})+}_{ij}, \phi^{(s;w)+}_{ij}, \phi^{(r;w)+}_{ij}$ and $\phi^{(\tau)+}_{ij}$

$[\phi^{(r;\mathfrak{V})}_{ij}]^{+}$; (present Green's function)
$[\phi^{(r;w)}_{ij}]^{+}$; (present Green's function)
$[\phi^{(s;\mathfrak{V})}_{ij}]^{+}$; (present Green's function)
$[\phi^{(s;w)}_{ij}]^{+}$; (present Green's function)
$[\phi^{\tau}_{ij}]^{+}$; (present Green's function)
$[\phi^{(r;\mathfrak{V})}_{ij}]^{+} + [\phi^{(r;w)}_{ij}]^{+} + [\phi^{(s;\mathfrak{V})}_{ij}]^{+} + [\phi^{(s;w)}_{ij}]^{+} + [\phi^{\tau}_{ij}]^{+}$; (present compressible DNS)
$[\phi_{ij}]^{+}$; (present compressible DNS)
○ $[\phi_{ij}]^{+}$; (Kim et al., 1987; incompressible DNS)
□ $[\phi_{ij}]^{+}$; (Hoyas and Jiménez, 2006; incompressible DNS)





- Conclusion

• The Tensor Representation of the pressure-velocity correlations Analysis shows us :
⇒ We have to improve the Redistribution tensor ϕ_{ij} to obtain a best prediction of the wall-bounded turbulent flows

• The original splitting method using the two Green's functions gives us information about the wall-echo terms

- Perspectives

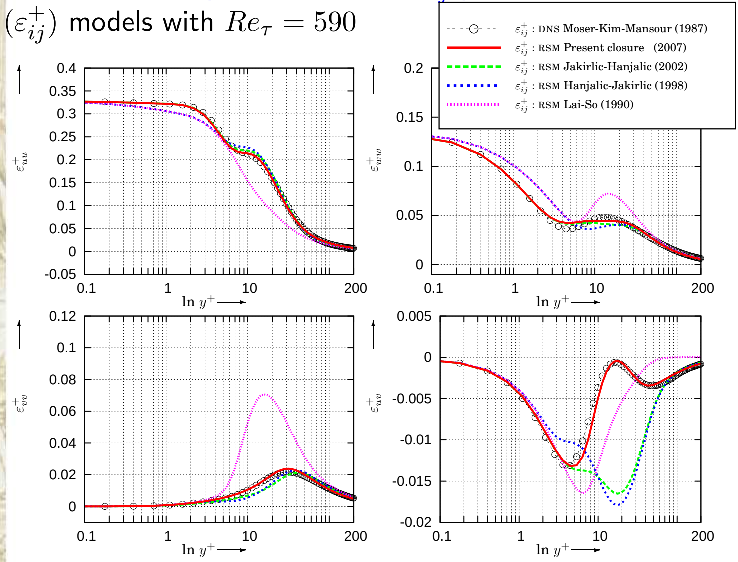
• The next Research fields will be to compare the volume terms and wall-echo terms from the DNS database with the Tensor Repesentations according to the Literature

• We have already developed an efficient Tensor Representation of the dissipation term ε_{ij}

• December 2007 : Setting up an on-line DNS database

- Tensor Representations for ε_{ij} present in the Literature

(ε_{ij}^{+}) models with $Re_\tau = 590$



- Favre-Reynolds-averaged Reynolds-Stress equations

$$\underbrace{\frac{\partial \bar{\rho}\widetilde{u''_i u''_j}}{\partial t} + \frac{\partial}{\partial x_\ell}(\bar{\rho}\widetilde{u''_i u''_j}\tilde{u}_\ell)}_{\text{convection } C_{ij}} = \underbrace{\frac{\partial}{\partial x_\ell}(-\bar{\rho}\widetilde{u''_i u''_j u''_\ell} - \overline{p' u''_j}\delta_{i\ell} - \overline{p' u''_i}\delta_{j\ell} + \overline{u''_i \tau'_{j\ell} + u''_j \tau'_{i\ell}})}_{\text{diffusion } d_{ij} = d^{(u)}_{ij} + d^{(p)}_{ij} + d^{(\mu)}_{ij}}$$

$$+ \underbrace{\overline{p'\left(\frac{\partial u''_i}{\partial x_j} + \frac{\partial u''_j}{\partial x_i} - \frac{2}{3}\frac{\partial u''_k}{\partial x_k}\delta_{ij}\right)}}_{\text{redistribution } \phi_{ij}} + \underbrace{\left(-\bar{\rho}\widetilde{u''_i u''_\ell}\frac{\partial \tilde{u}_j}{\partial x_\ell} - \bar{\rho}\widetilde{u''_j u''_\ell}\frac{\partial \tilde{u}_i}{\partial x_\ell}\right)}_{\text{production } P_{ij}} - \underbrace{\overline{\left(\tau'_{j\ell}\frac{\partial u''_i}{\partial x_\ell} + \tau'_{i\ell}\frac{\partial u''_j}{\partial x_\ell}\right)}}_{\text{dissipation } \bar{\rho}\varepsilon_{ij}}$$

$$+ \underbrace{\frac{2}{3}\overline{p'\frac{\partial u''_k}{\partial x_k}}\delta_{ij}}_{\text{pressure-dilatation } \phi_p} + \underbrace{\left(-\overline{u''_i}\frac{\partial \bar{p}}{\partial x_j} - \overline{u''_j}\frac{\partial \bar{p}}{\partial x_i} + \overline{u''_i}\frac{\partial \bar{\tau}_{j\ell}}{\partial x_\ell} + \overline{u''_j}\frac{\partial \bar{\tau}_{i\ell}}{\partial x_\ell}\right)}_{\text{density fluctuation effects } K_{ij}} + \underbrace{\left(\overline{\rho u''_j f'_{V_i} + \rho u''_i f'_{V_j}}\right)}_{\text{body force effects } B_{ij}}$$

- ϕ_{ij} and $d^{(p)}_{ij}$ are both predominant terms in RSM closures

$\Longrightarrow$ The redistribution term ϕ_{ij} <u>is not properly modelled</u>

• The Poisson Equation on the Splitting form :

$$\nabla^2 p'_m = Q'_m \quad (1) \quad ; \quad \frac{\partial p'_m}{\partial y} = B'_{(m)\pm}(x,z,t) \quad : \quad y = \pm\tfrac{1}{2}L_y$$

with $m = [(\tau),(\mathrm{BF}),(s),(r)]$ for each term (Stokes, Body-Force, slow and rapid)

the system (1) is reduced to a (ODE) in the Fourier domain :

$$\left[\frac{d^2}{dy^2} - \kappa^2\right]\hat{p}'_{(m)}(\kappa_x,y,\kappa_z,t) = \hat{Q}'_{(m)}(\kappa_x,y,\kappa_z,t) \quad (2)$$

$$\kappa^2 := \kappa_x^2 + \kappa_z^2$$

with:

$$p'_{(m)}(x,y,z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{p}'_{(m)}(\kappa_x,y,\kappa_z,t)e^{i\kappa_x x + i\kappa_z z}d\kappa_x d\kappa_z$$

$$Q'_{(m)}(x,y,z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{Q}'_{(m)}(\kappa_x,y,\kappa_z,t)e^{i\kappa_x x + i\kappa_z z}d\kappa_x d\kappa_z$$

$$B'_{(m)}(x,y,z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{B}'_{(m)}(\kappa_x,\kappa_z,t)e^{i\kappa_x x + i\kappa_z z}d\kappa_x d\kappa_z$$

Notice that for the plane channel flow $f_{\mathrm{V}_y} \equiv 0.$ so that only $B'_{(\tau)\pm} \neq 0$

$\Rightarrow$ other terms having Neumann bondary-conditions $B'_{(m)\pm} = 0 \;\; ; \;\; \forall(m) \neq (\tau)$

- The solutions of the Green Functions

When $\kappa \neq 0$ the Solutions are :

$$G_{\text{Kim}}(\kappa \neq 0, y, Y) = -\frac{\cosh[\kappa(L_y - |y - Y|)] + \cosh[\kappa(y+Y)]}{2\kappa \sinh \kappa L_y}$$

$$\hat{p}'_{(m)_{\text{BCs}}}(\kappa_x, y, \kappa_z, t) = \frac{B'_{(m)_+} \cosh[\kappa(\frac{1}{2}L_y + y)] - B'_{(m)_-} \cosh[\kappa(\frac{1}{2}L_y - y)]}{\kappa \sinh \kappa L_y} \quad ; \quad \kappa := \sqrt{\kappa_x^2 + \kappa_z^2} \neq 0$$

$$G_{\infty}(\kappa \neq 0, y, Y) = -\frac{e^{-\kappa|y-Y|}}{2\kappa}$$

When $\kappa = 0$ the Solutions are :

$$G_{\text{Kim}}(\kappa = 0, y, Y) = G_{\infty}(\kappa = 0, y, Y) = \frac{|y - Y|}{2}$$

$$\hat{p}'_{(m)_{\text{BCs}}}(\kappa_x = 0, y, \kappa_z = 0, t) = 0$$

$$\hat{p}'_{(m;\mathfrak{V})}(\kappa_x = 0, y, \kappa_z = 0, t) = \hat{p}'_{(m)}(\kappa_x = 0, y, \kappa_z = 0, t)$$

$$\iff \quad \hat{p}'_{(m;w)}(\kappa_x = 0, y, \kappa_z = 0, t) = 0$$

- $\phi^{+}_{ij}, \phi^{(\mathrm{s};\mathfrak{V})+}_{ij}, \phi^{(\mathrm{r};\mathfrak{V})+}_{ij}, \phi^{(\mathrm{s};w)+}_{ij}, \phi^{(\mathrm{r};w)+}_{ij}$ and $\phi^{(\tau)+}_{ij}$

$[\phi^{(\mathrm{r};\mathfrak{V})}_{ij}]^{+}$; (present Green's function)
$[\phi^{(\mathrm{r};w)}_{ij}]^{+}$; (present Green's function)
$[\phi^{(\mathrm{s};\mathfrak{V})}_{ij}]^{+}$; (present Green's function)
$[\phi^{(\mathrm{s};w)}_{ij}]^{+}$; (present Green's function)
$[\phi^{\tau}_{ij}]^{+}$; (present Green's function)
$[\phi^{(\mathrm{r};\mathfrak{V})}_{ij}]^{+} + [\phi^{(\mathrm{r};w)}_{ij}]^{+} + [\phi^{(\mathrm{s};\mathfrak{V})}_{ij}]^{+} + [\phi^{(\mathrm{s};w)}_{ij}]^{+} + [\phi^{\tau}_{ij}]^{+}$; (present compressible DNS)
$[\phi_{ij}]^{+}$; (present compressible DNS)
○ $[\phi_{ij}]^{+}$; (Kim et al., 1987; incompressible DNS)
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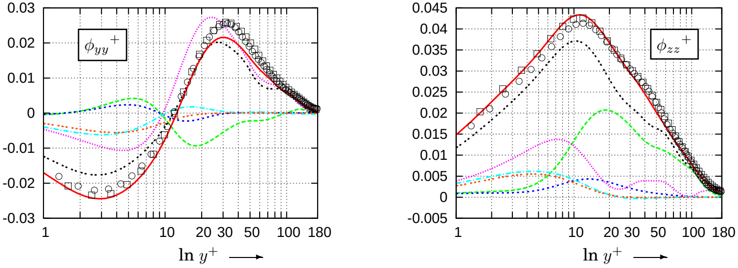

- Numerical parameters of the DNS computation
  - Isothermal walls
  - High-order upwind scheme using the $O(\Delta x_H^9)$ reconstitution with the HLLC ARS
  - no limiters
  - Implicit $O(\Delta t^2)$ time-integration with explicit subiterations [5 subit.]

Table: Summary of DNS computations.

Re_{τ_w}	$\bar{M}_{\mathrm{B}_w}$	$\bar{M}_{\mathrm{CL}}$	$L_x \times L_y \times L_z$	$N_x \times N_y \times N_z$	$\overline{\Delta x^+}$	$\overline{\Delta y_w^+}$	$\overline{\Delta z^+}$	scheme
182	0.300	0.341	$4\pi\delta \times 2\delta \times \frac{4}{3}\pi\delta$	$57 \times 121 \times 49$	52	0.2	20	UW9
185	0.300	0.347	$4\pi\delta \times 2\delta \times \frac{4}{3}\pi\delta$	$57 \times 161 \times 49$	52	0.2	20	UW9
185	0.300	0.347	$4\pi\delta \times 2\delta \times \frac{4}{3}\pi\delta$	$121 \times 121 \times 81$	24	0.2	12	UW9
239	1.507	1.492	$4\pi\delta \times 2\delta \times \frac{4}{3}\pi\delta$	$57 \times 161 \times 49$	52	0.2	20	UW9
239	1.507	1.502	$4\pi\delta \times 2\delta \times \frac{4}{3}\pi\delta$	$121 \times 121 \times 81$	24	0.2	12	UW9