ETC11, Porto, 25-28 June 2007

26.06. Room 3. Session 3.4 Wall-bounded flows (IV), 10:25-10:45.

Master-mode set for 3D turbulent channel flow

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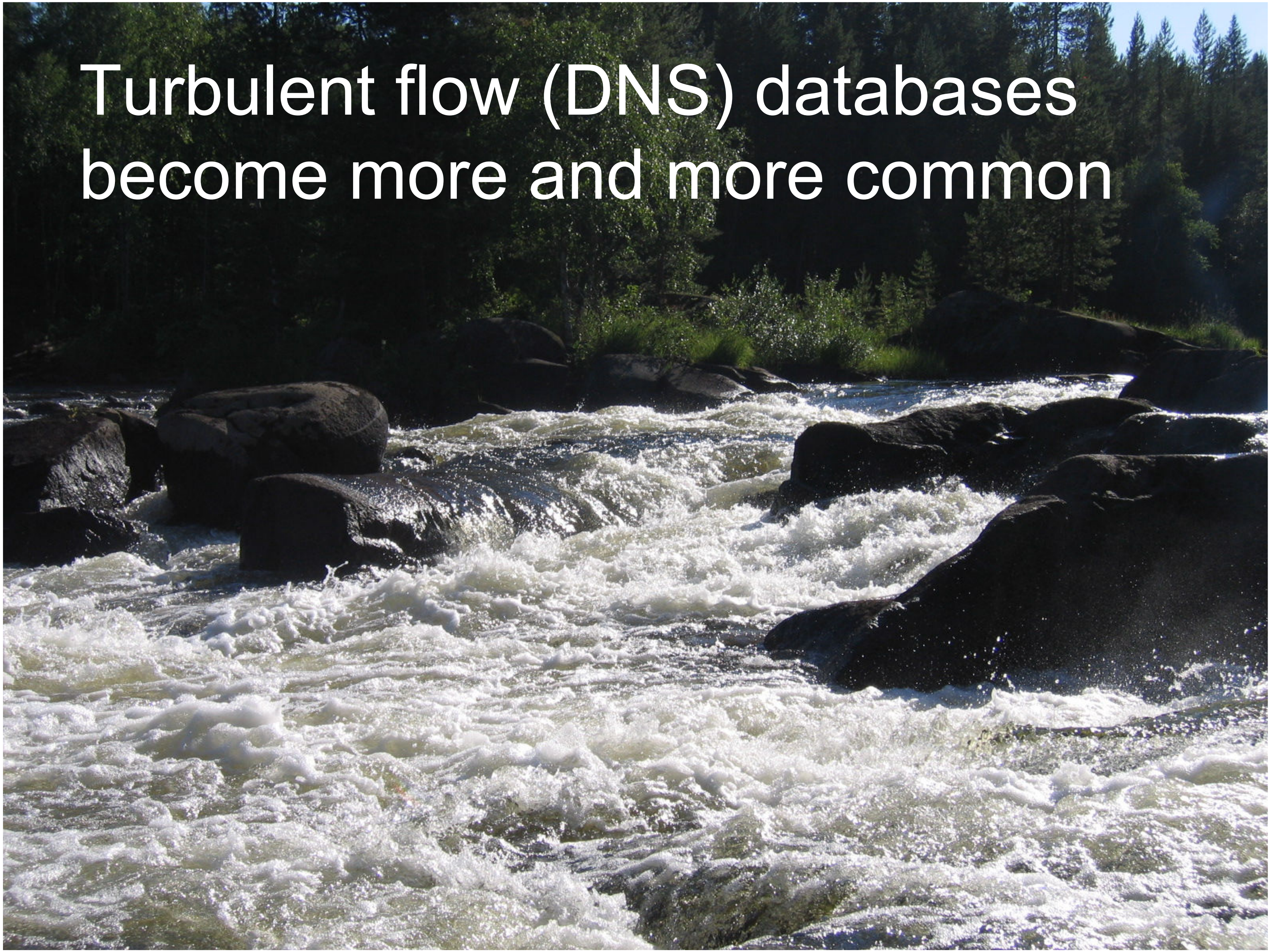
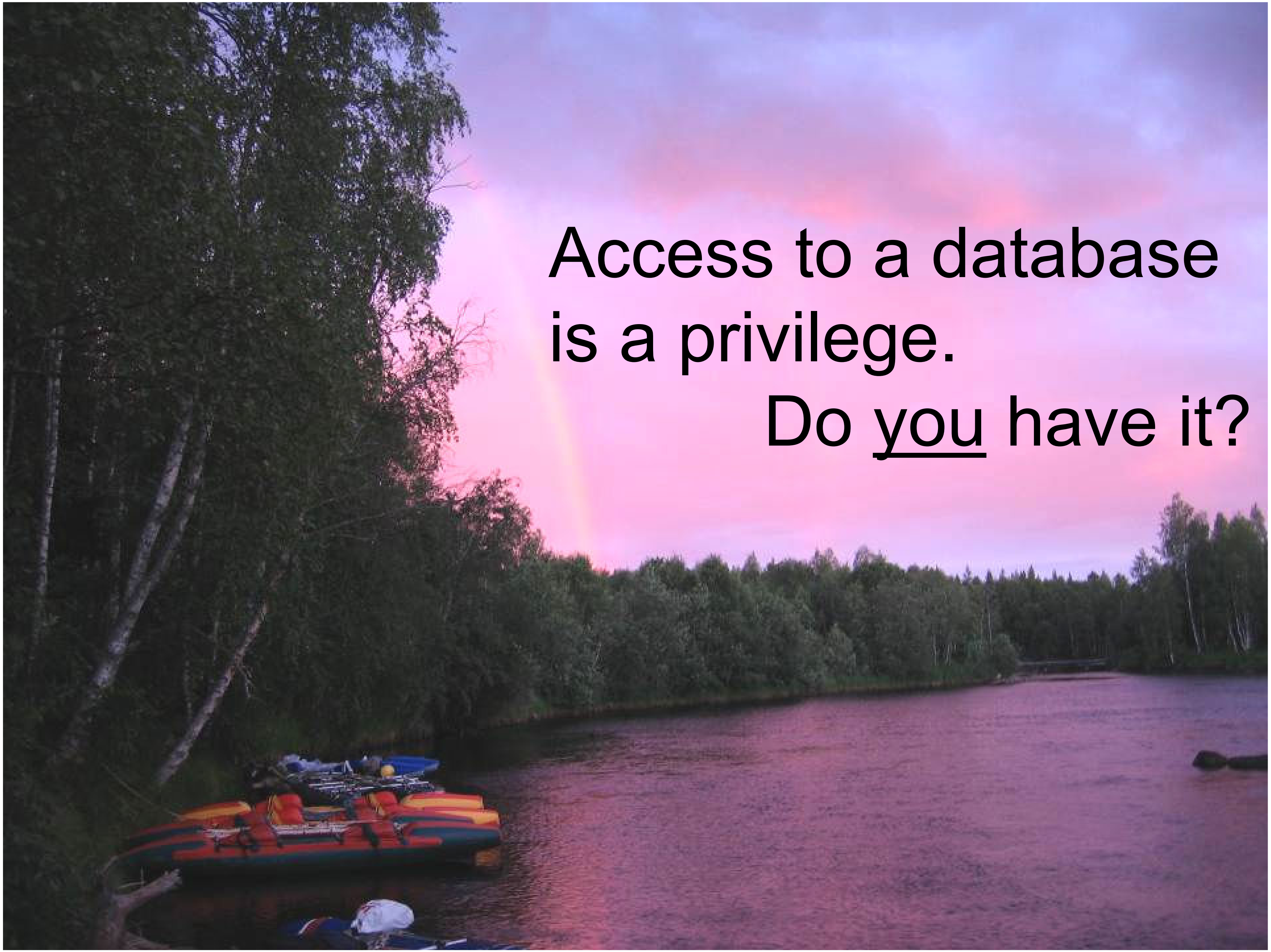
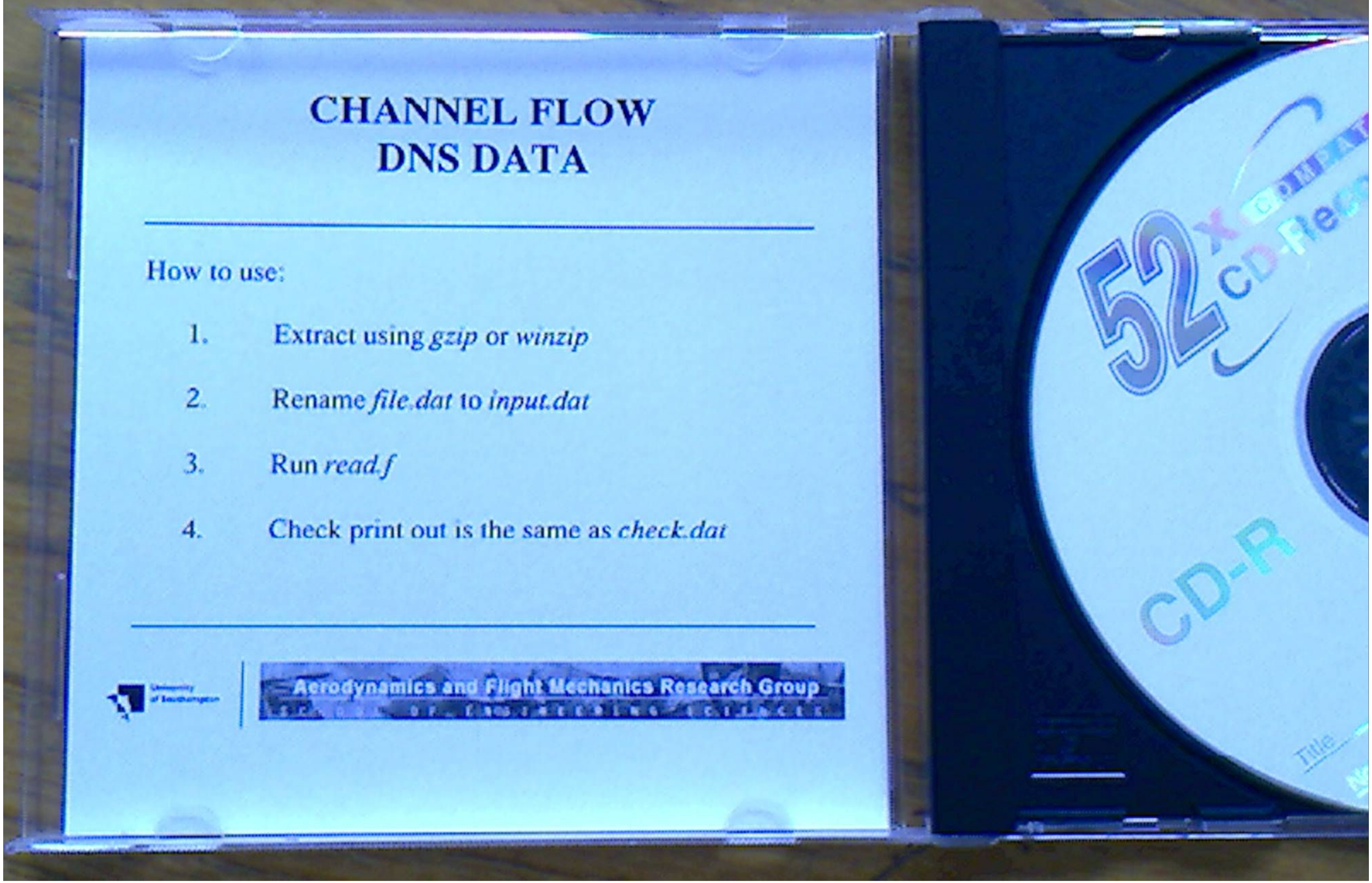

Turbulent flow (DNS) databases
become more and more common

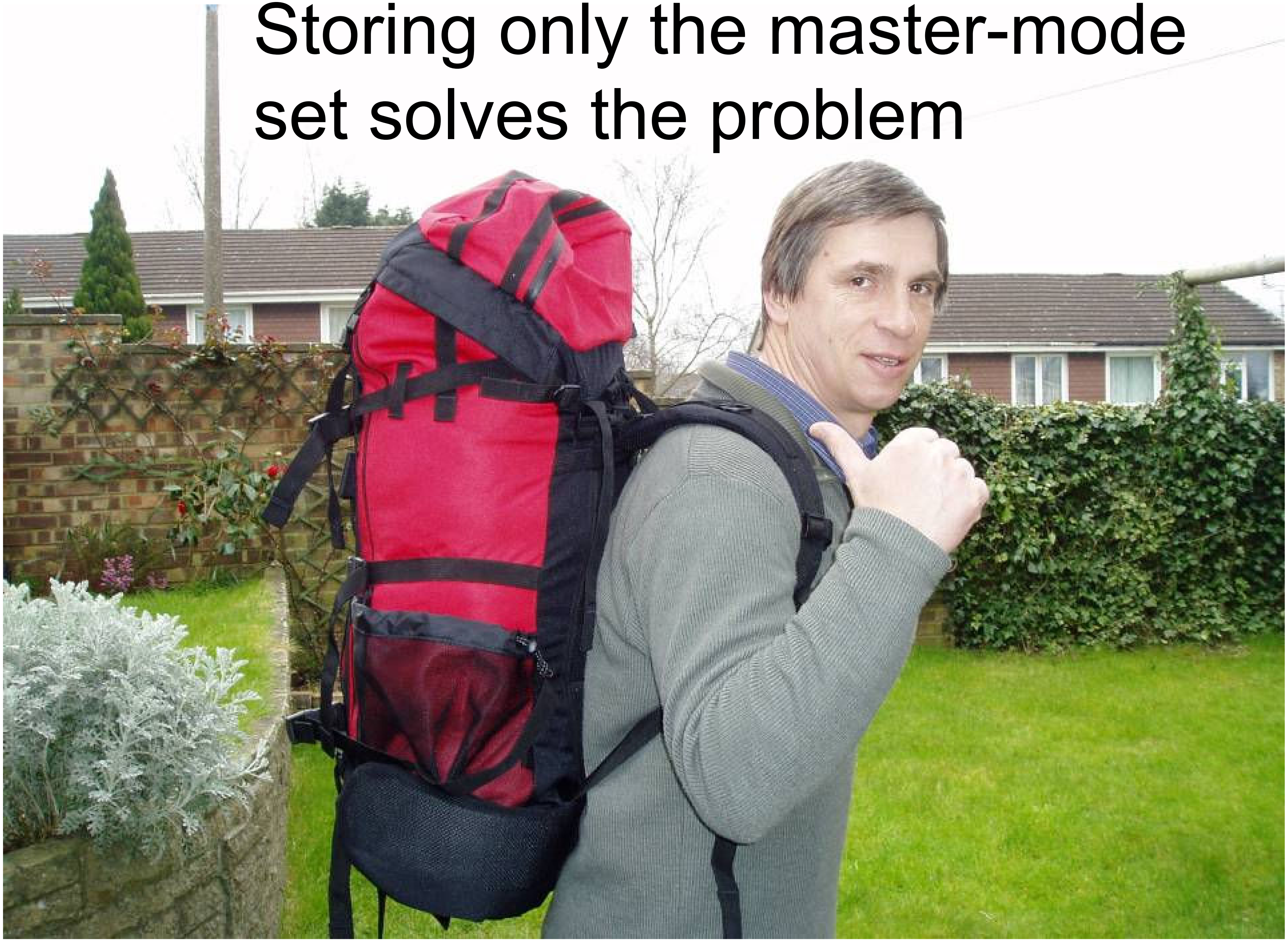

Access to a database
is a privilege.
Do you have it?

Full DNS database requires too much storage

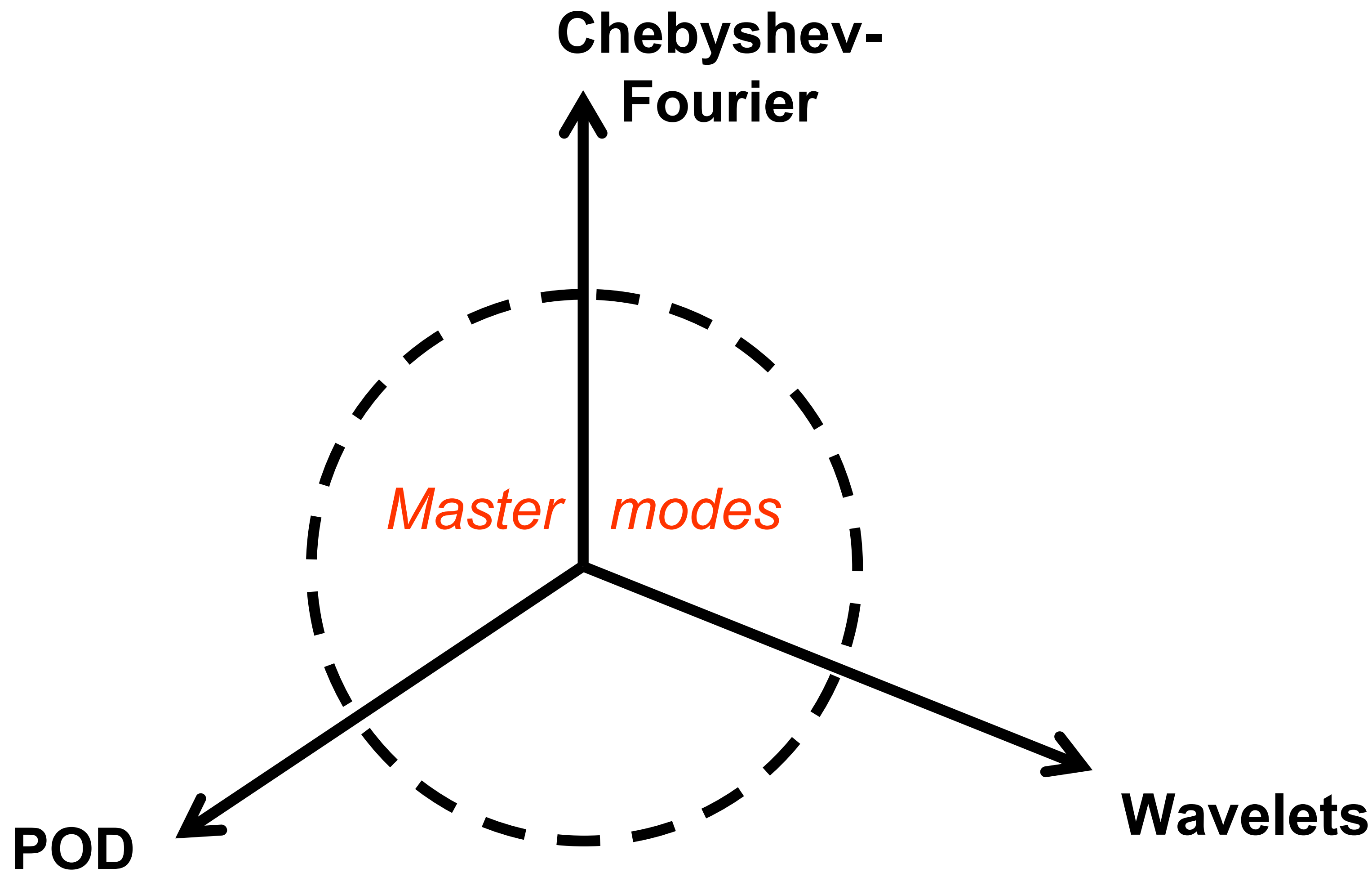
Complete database can be small



Storing only the master-mode set solves the problem

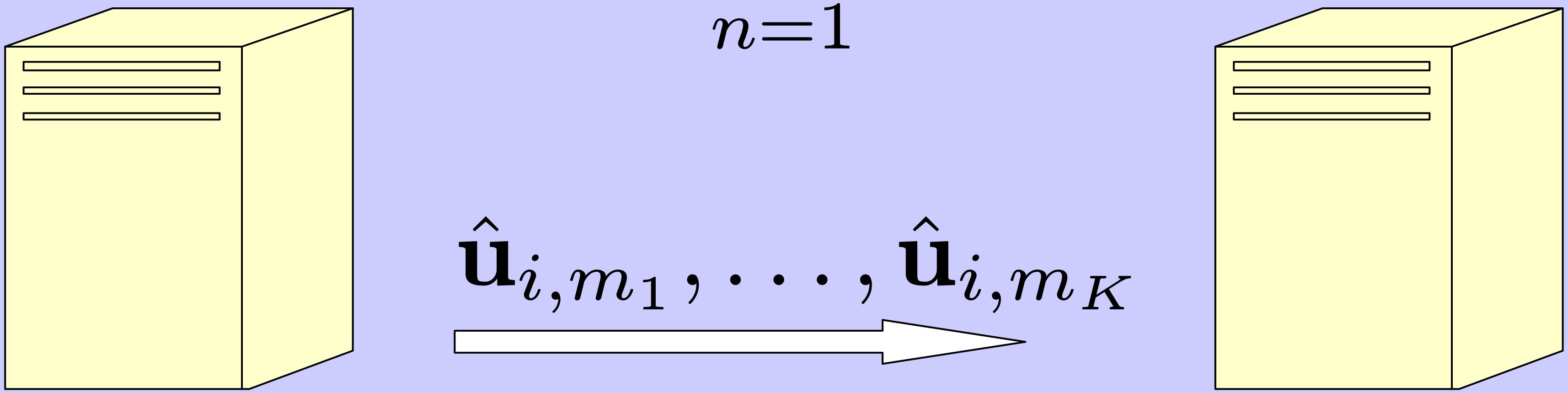


What master-modes are not



Master code dictates master-modes to the slave code

$$\mathbf{u}_i(\mathbf{x}) = \sum_{n=1}^{S} \hat{\mathbf{u}}_{in} \boldsymbol{\phi}_n(\mathbf{x}), \;\; t = t_i$$



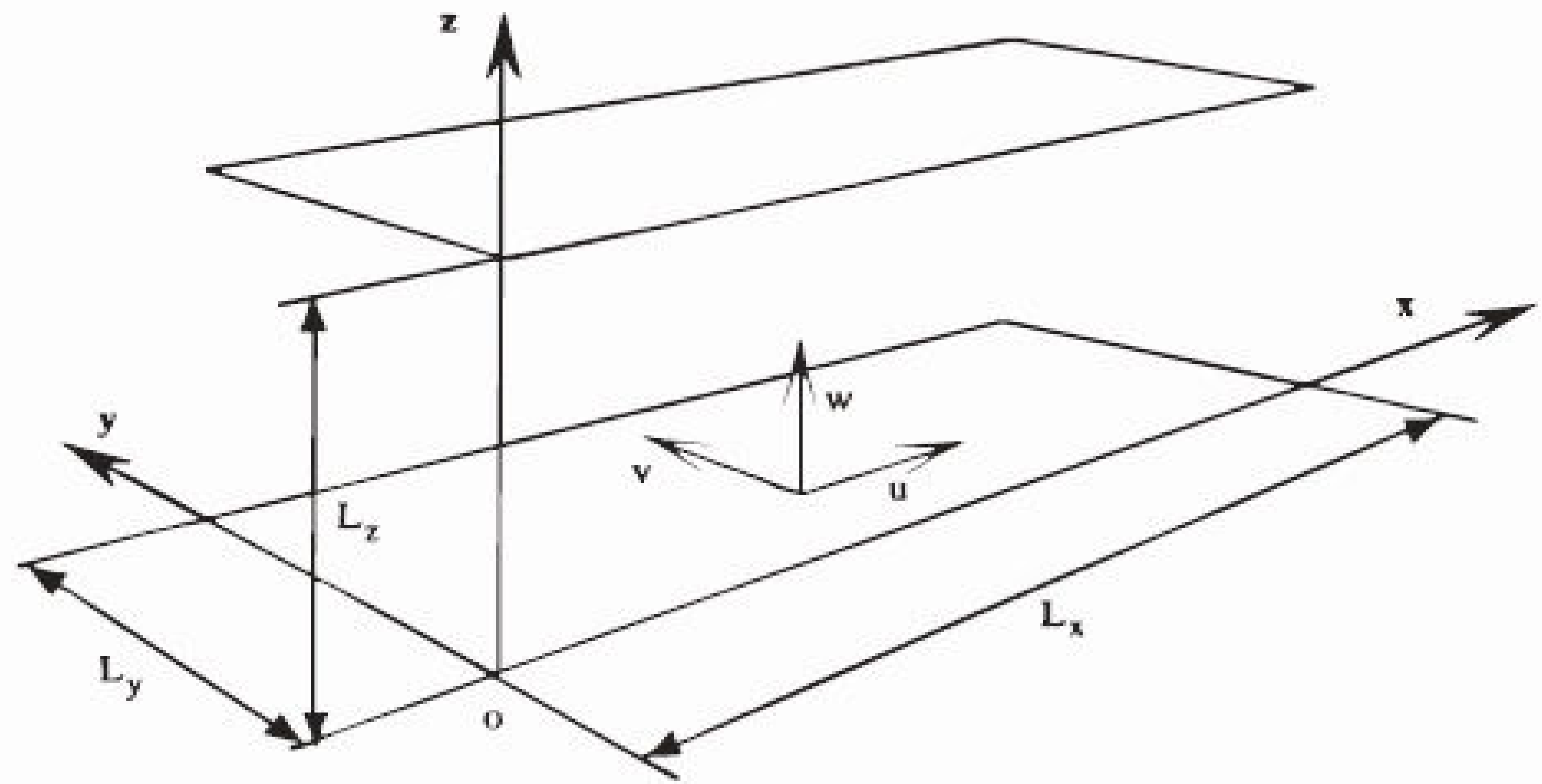
$$\hat{\mathbf{u}}_{i,m_1}, \ldots, \hat{\mathbf{u}}_{i,m_K}$$

Master code

Slave code

$$\boxed{||\mathbf{u}_{\text{master}} - \mathbf{u}_{\text{slave}}|| \overset{?}{\rightarrow} 0}$$

3D channel flow has a master-mode set

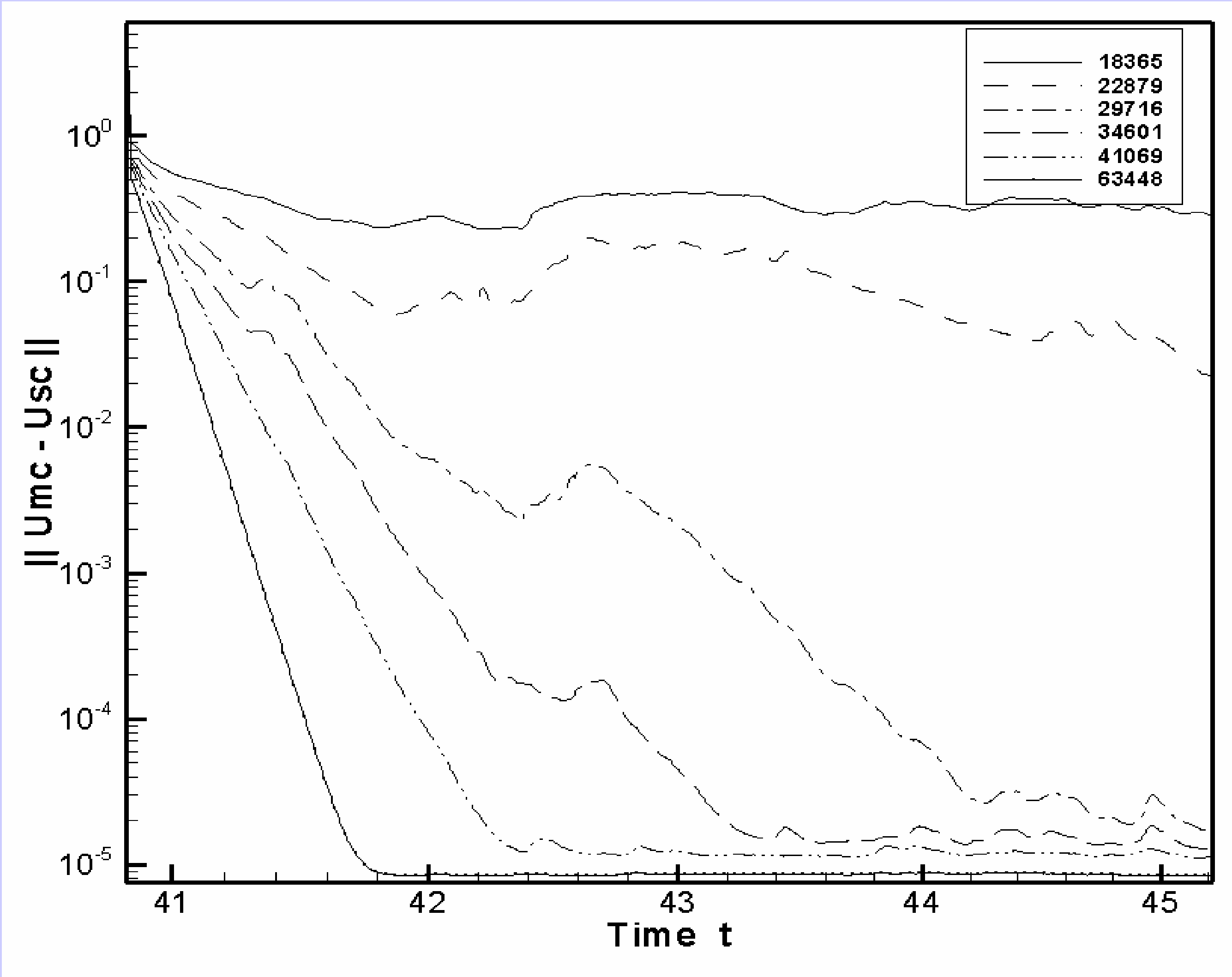


Our code works

- Sandham and Howard pseudo-spectral code is reliable
- We test codes by adding a body force
- Standard comparisons were also made

Slave solution converges to the master solution

$Re_\tau = 360$

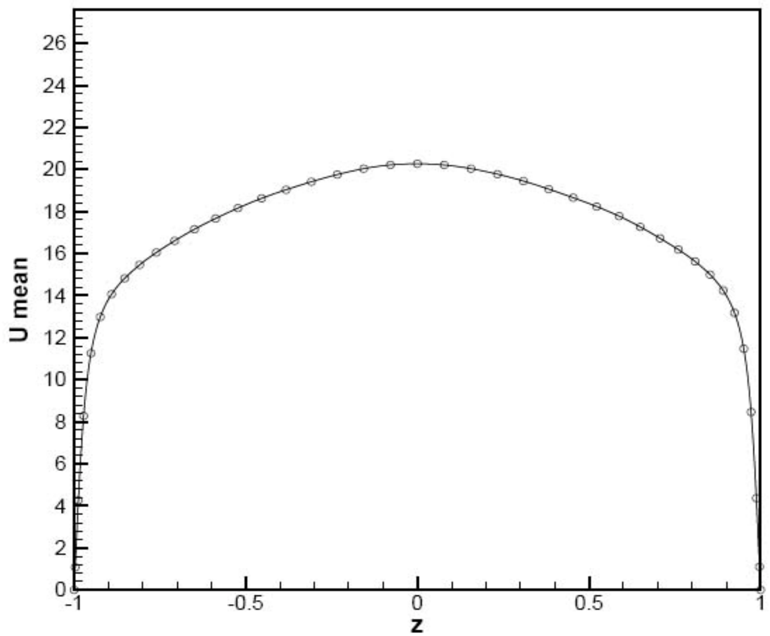


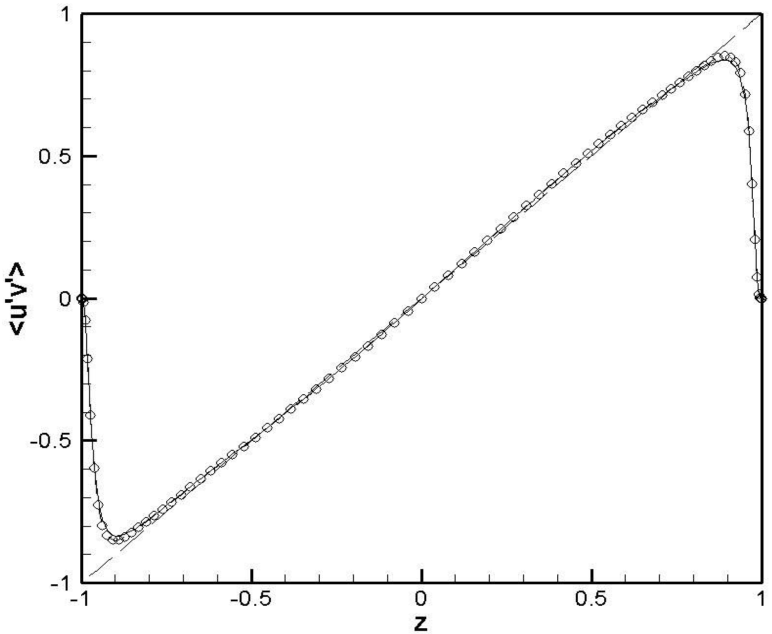
$6 \times 3 \times 2$ box $\qquad K \sim 30000$

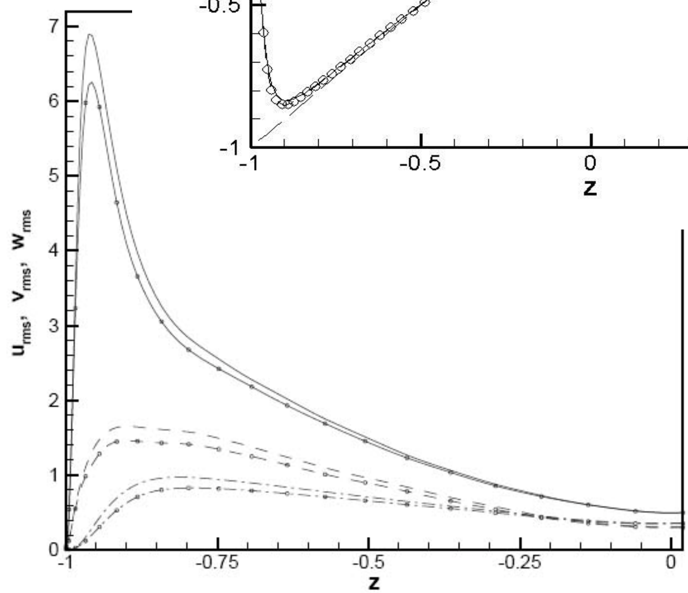
Master-mode-based database is 100 times smaller

- We had 2621440 modes in total
- Master-mode set size is less than 30000
- ~1%!

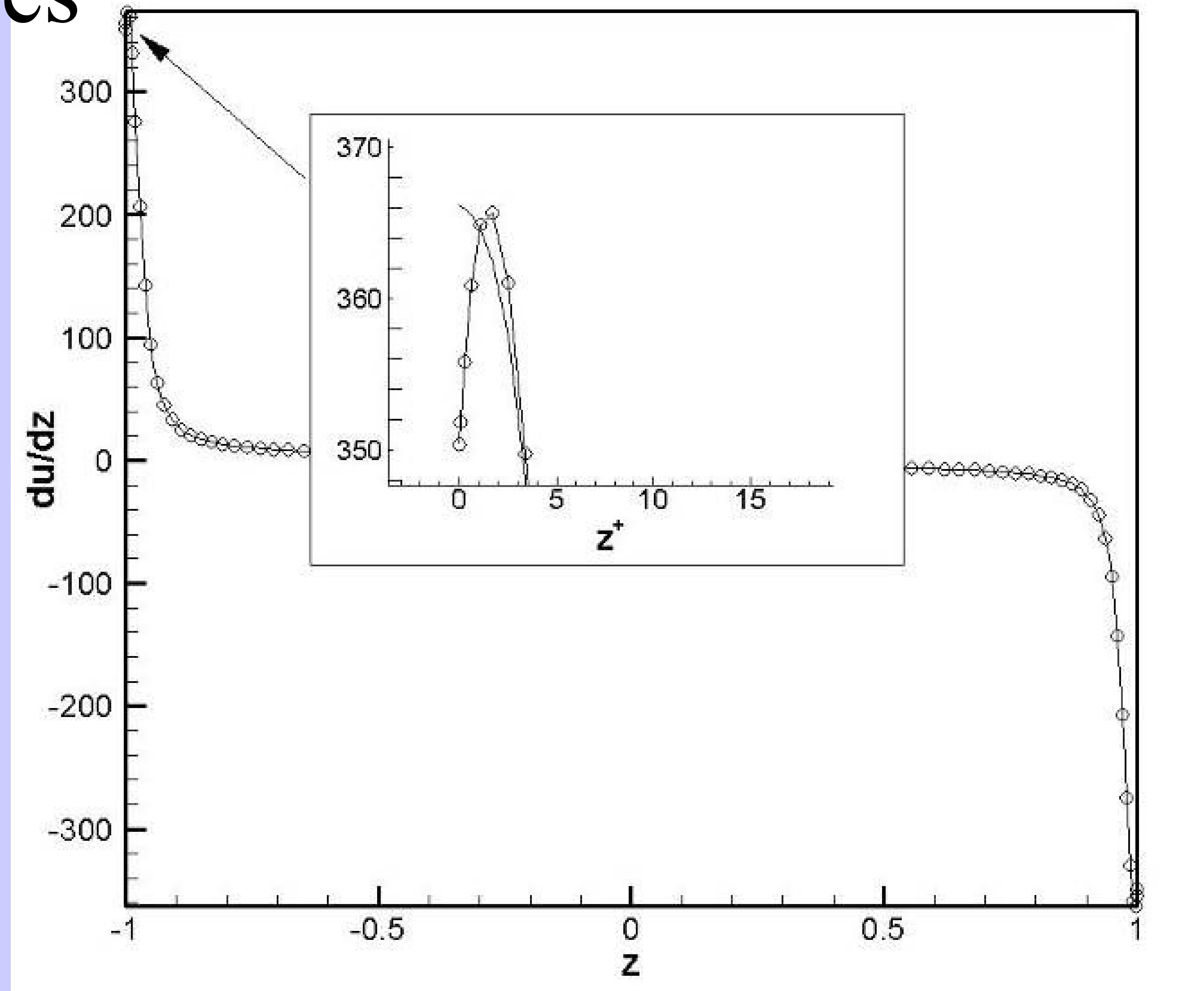
Master-modes alone provide a decent approximation



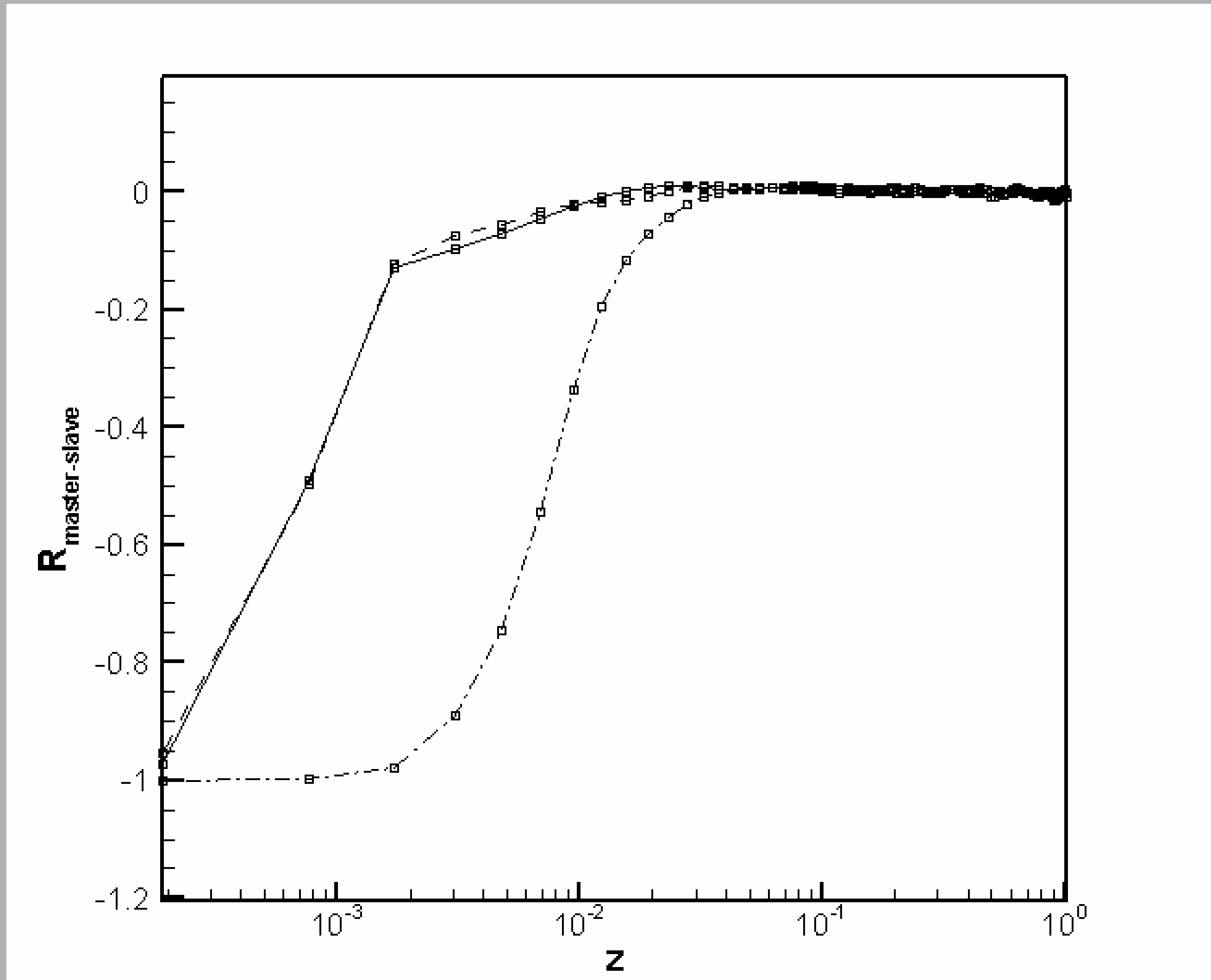




Near the wall there is a problem with derivatives

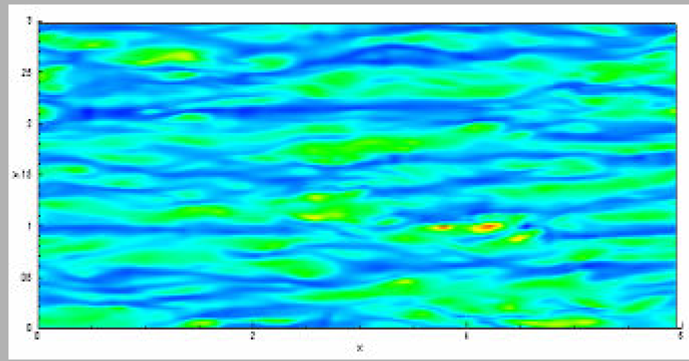


Near the wall master-slave correlation is high

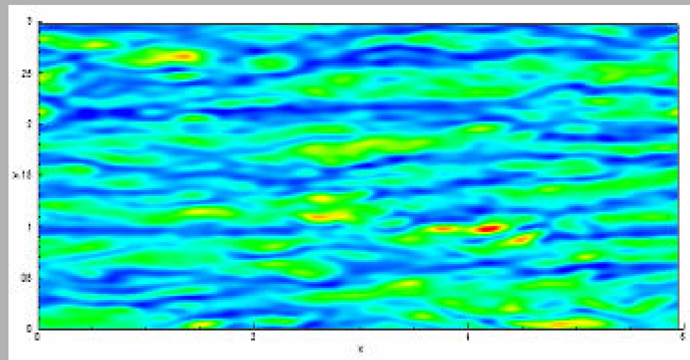


Streaks are in the master-mode set

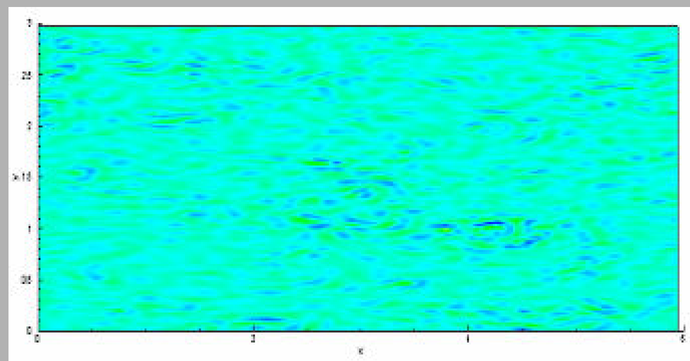
Full



Master

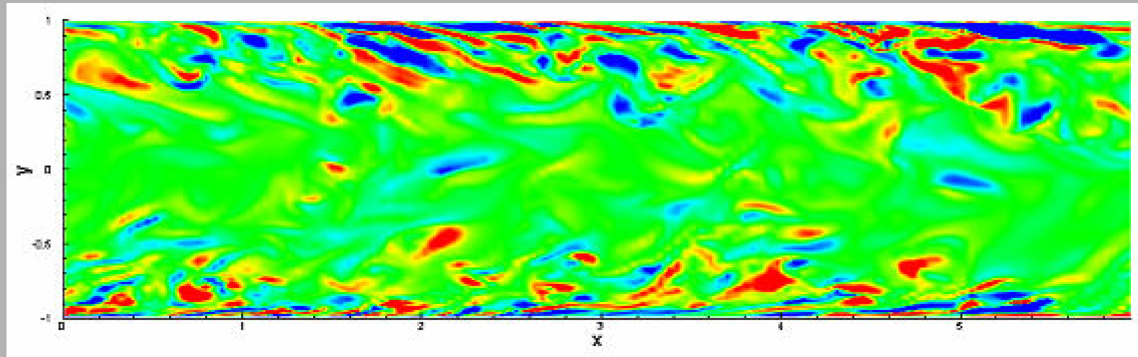


Slave

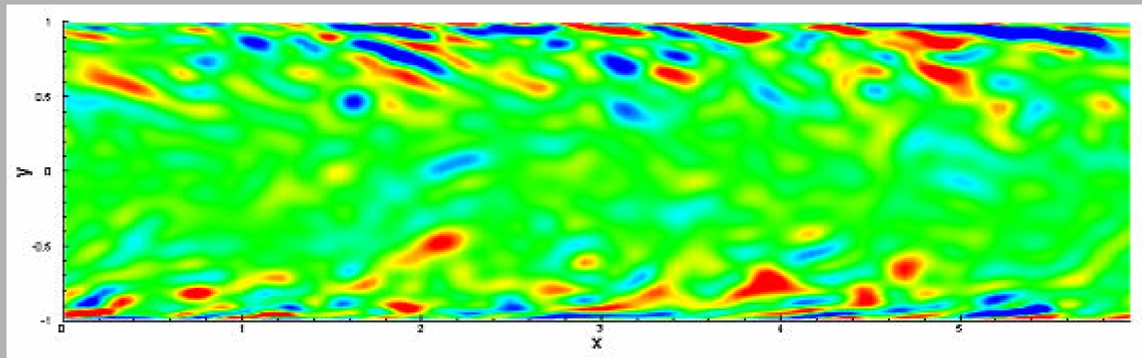


Vortices are mostly in master-mode set

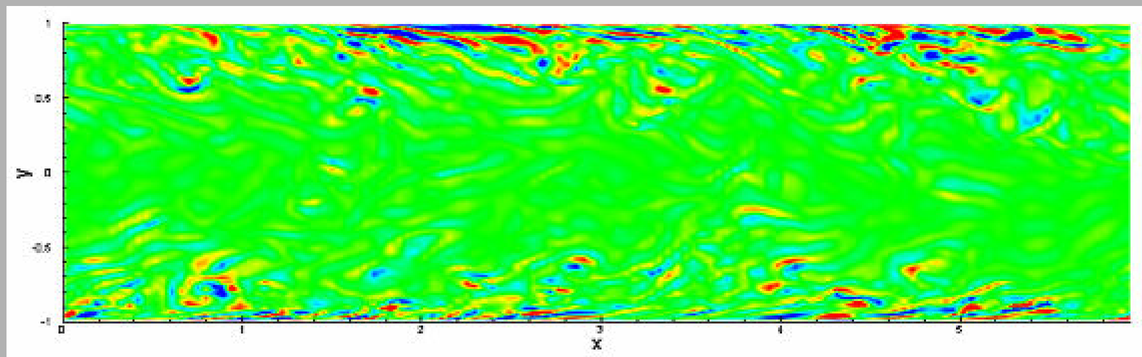
Full



Master



Slave

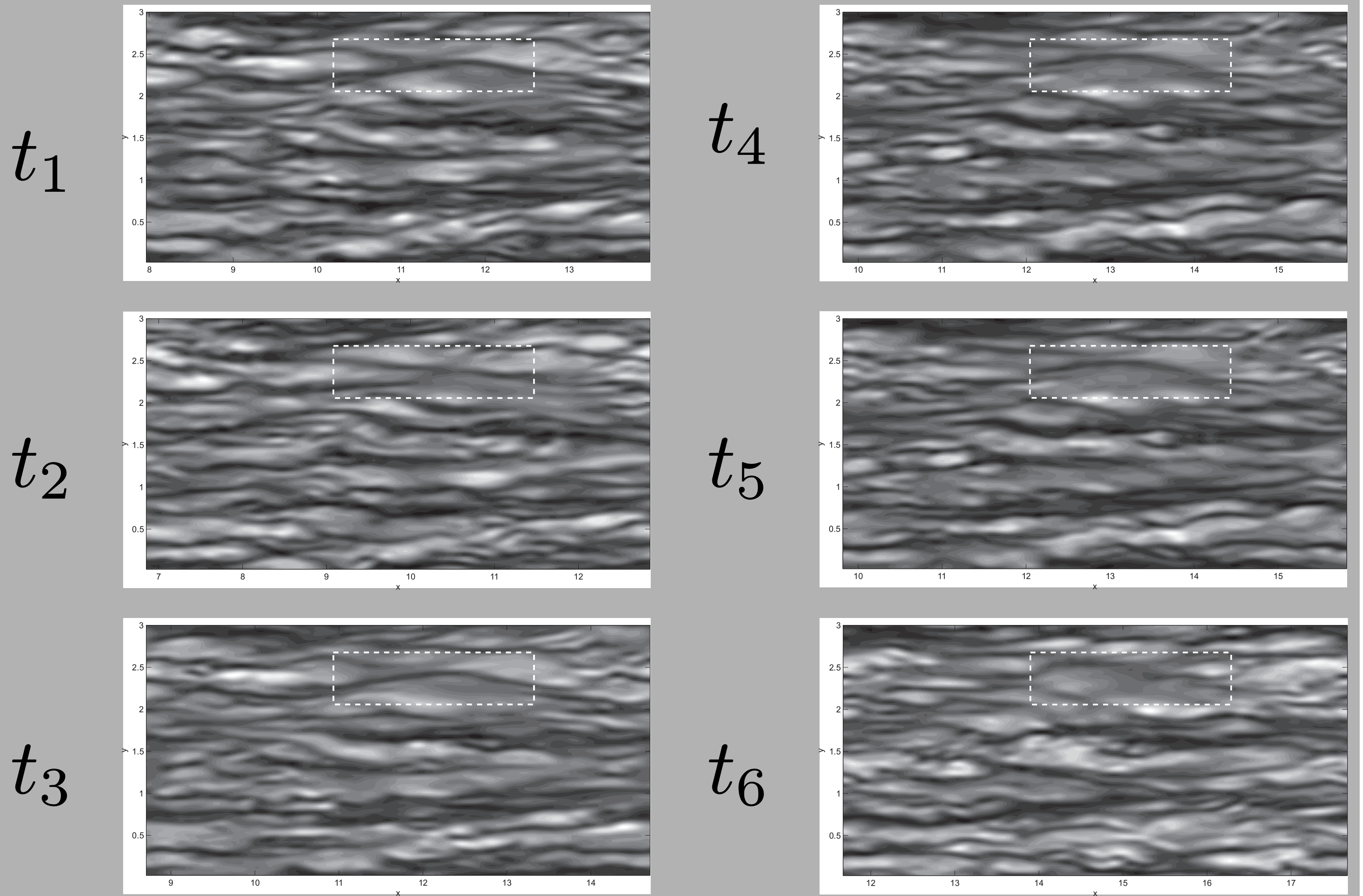


Master-mode database is best suited for catching rare events

$$\mathbf{u}(x, y, z, t) = \mathbf{U}(x - ct, y, z)$$

$$\min_{c} \iiint\limits_{V} \sum_{i=1}^{3} \left(\frac{\partial u_i}{\partial t} + c \frac{\partial u_i}{\partial x} \right)^2 \approx 0$$

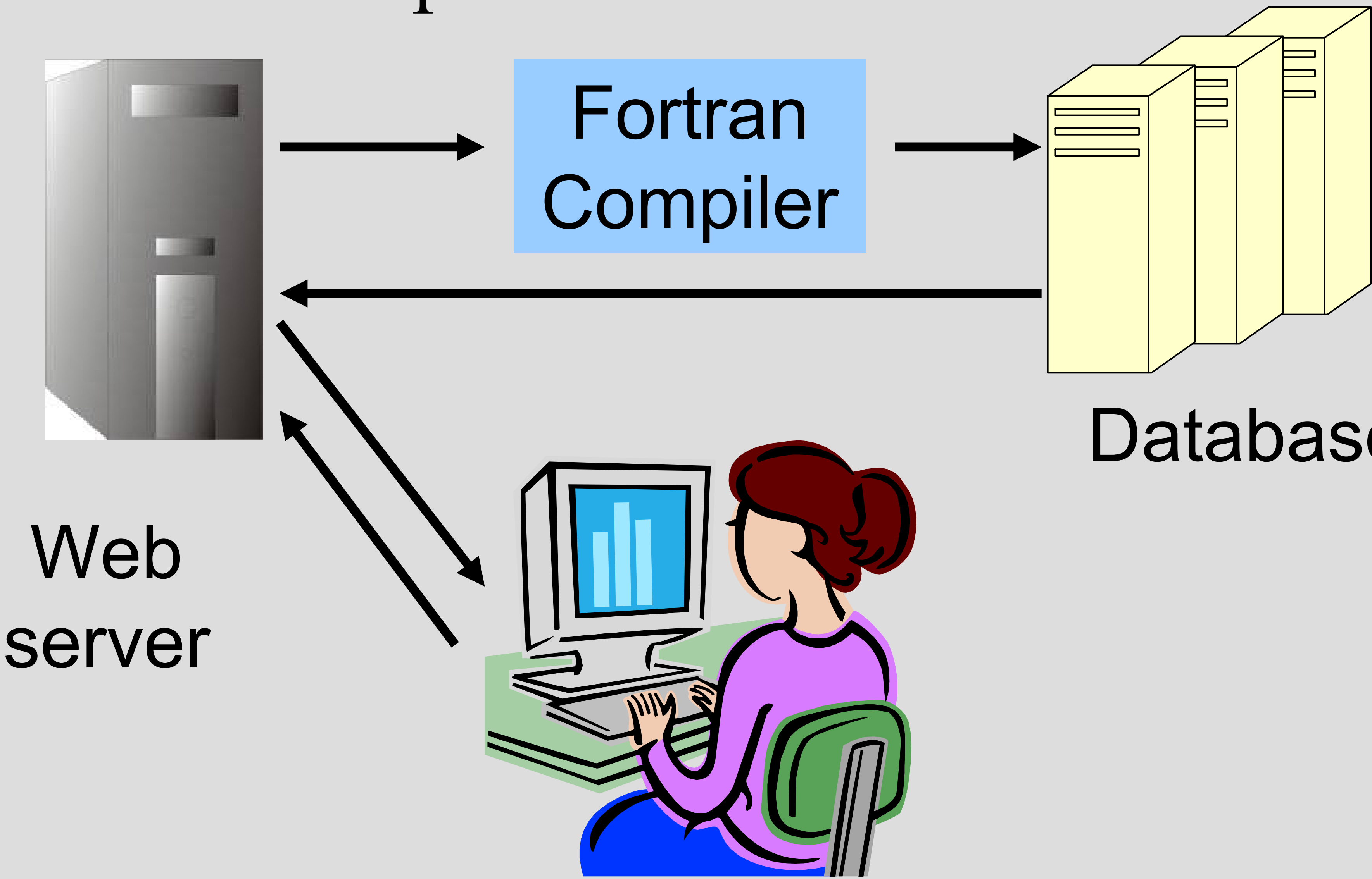
Travelling-wave-like object is detected



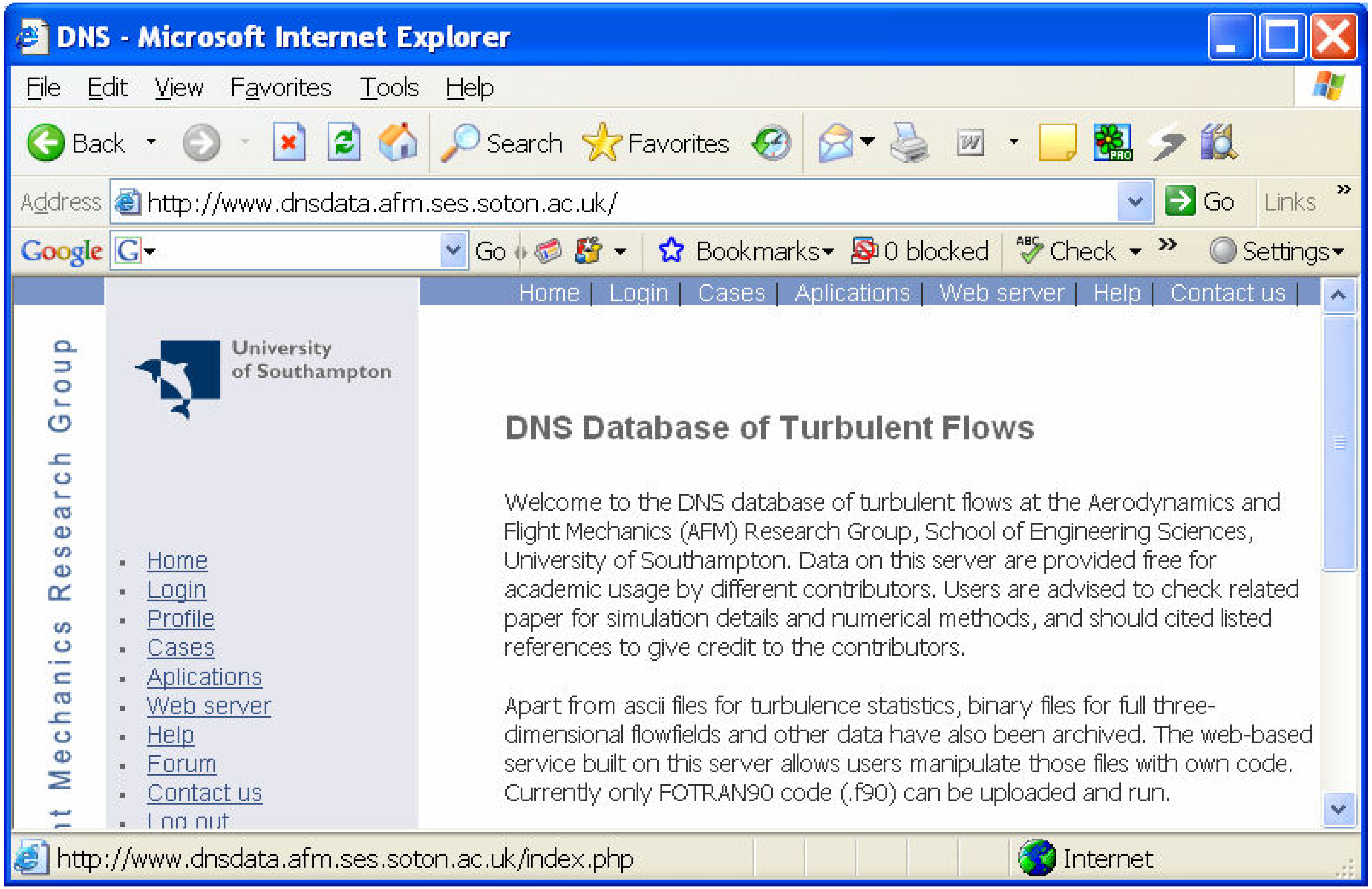
Can you have access to the database?
Right now, today?


One could use a full database but
they are too large!

Storing only the master-mode set solves the problem



Master-mode database is online at www.dnsdata.afm.ses.soton.ac.uk




Join us online in using master-modes
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Additional slides

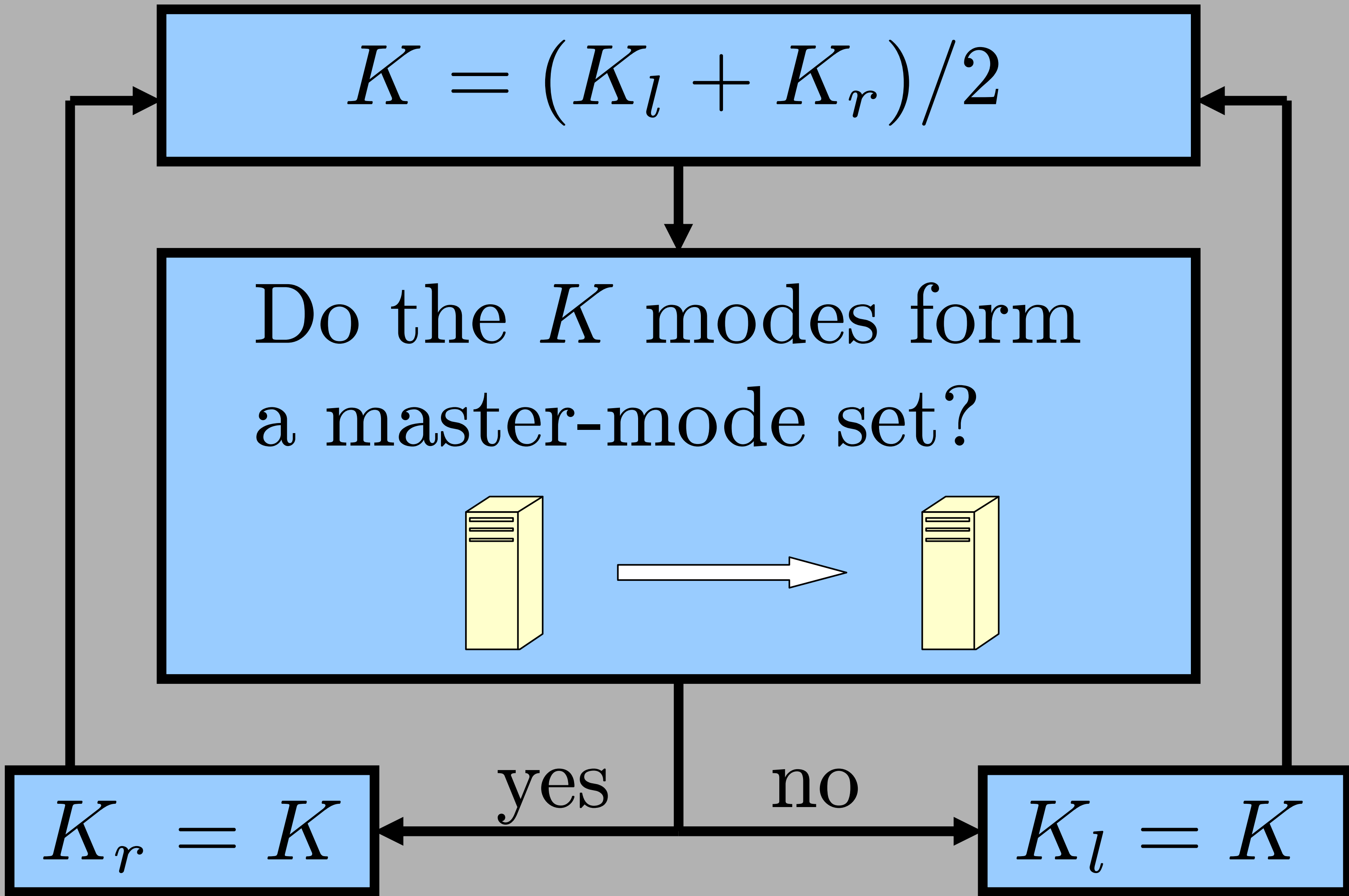
Master-mode set determines the entire solution

$$\left.\begin{aligned} \mathbf{u}_A &= \sum_{n=1}^{\infty} A_n(t)\boldsymbol{\phi}_n(\mathbf{x}), \\ \mathbf{u}_B &= \sum_{n=1}^{\infty} B_n(t)\boldsymbol{\phi}_n(\mathbf{x}) \end{aligned} \;\right|\; \begin{gathered} A_n(t) = B_n(t), \\ 1 \le n \le N \end{gathered}$$

$$\Downarrow$$

$$||\mathbf{u}_A - \mathbf{u}_B|| \to 0 \quad \text{as} \quad t \to \infty.$$

Dichotomy is faster than trial-and-error



Ordering by mean amplitude is best

$$Re_\tau = 180,\ L_x \times L_y \times L_z = 4 \times 3 \times 2$$

Method	
Wavenumber	K > 5000
Energy-based	2800 < K < 3500
Enstrophy-based	K~4800

Time marching is done by applying a recurrent formula to mode amplitudes

$$\mathbf{u}_i(\mathbf{x}) = \sum_{n=1}^{S} \hat{\mathbf{u}}_{in} \boldsymbol{\phi}_n(\mathbf{x}), \qquad t = t_i$$

$$\hat{\mathbf{u}}_{i+1,n} = D_n(\hat{\mathbf{u}}_{i,1}, \hat{\mathbf{u}}_{i,2}, \ldots, \hat{\mathbf{u}}_{i,S})$$

Master-mode set definition

$$\mathbf{v}_i(\mathbf{x}) = \sum_{n=1}^{S} \hat{\mathbf{v}}_{i,n} \boldsymbol{\phi}_n(\mathbf{x})$$

$$M = \{m_1, \ldots, m_K\}$$

$$\hat{\mathbf{v}}_{i+1,n} = \begin{cases} \hat{\mathbf{u}}_{i+1,n} & , \quad n \in M \\ D_n(\hat{\mathbf{v}}_{i,1}, \ldots, \hat{\mathbf{v}}_{i,K}), & n \notin M \end{cases}$$

$$||\mathbf{v}_i(\mathbf{x}) - \mathbf{u}_i(\mathbf{x})|| \rightarrow 0 \text{ as } i \rightarrow \infty \ \forall\, \mathbf{v}_1(\mathbf{x})$$

$\Leftrightarrow M$ is a master-mode set

Olson and Titi (2003) ordered modes by wavenumbers

$$\mathbf{u} = \sum_{k_x, k_y} \hat{\mathbf{u}}(t, k_x, k_y) e^{i(k_x x + k_y y)}$$

$$\boxed{k_x^2 + k_y^2 < \lambda}$$

Ordering by mean amplitude is similar to ordering by energy

$$\mathbf{u} = \sum_{k_x, k_y, k_z} \hat{\mathbf{u}}_{k_x, k_y, k_z}(t) e^{i(k_x x + k_y y)} T_{k_z}(z)$$

$$\boxed{\langle u^2_{k_x, k_y, k_z} \rangle > \lambda}$$

Can enstrophy-based ordering be better?

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}$$

$$\boldsymbol{\omega} = \sum_{k_x,k_y,k_z} \hat{\boldsymbol{\omega}}_{k_x,k_y,k_z}(t) e^{i(k_x x + k_y y)} T_{k_z}(z)$$

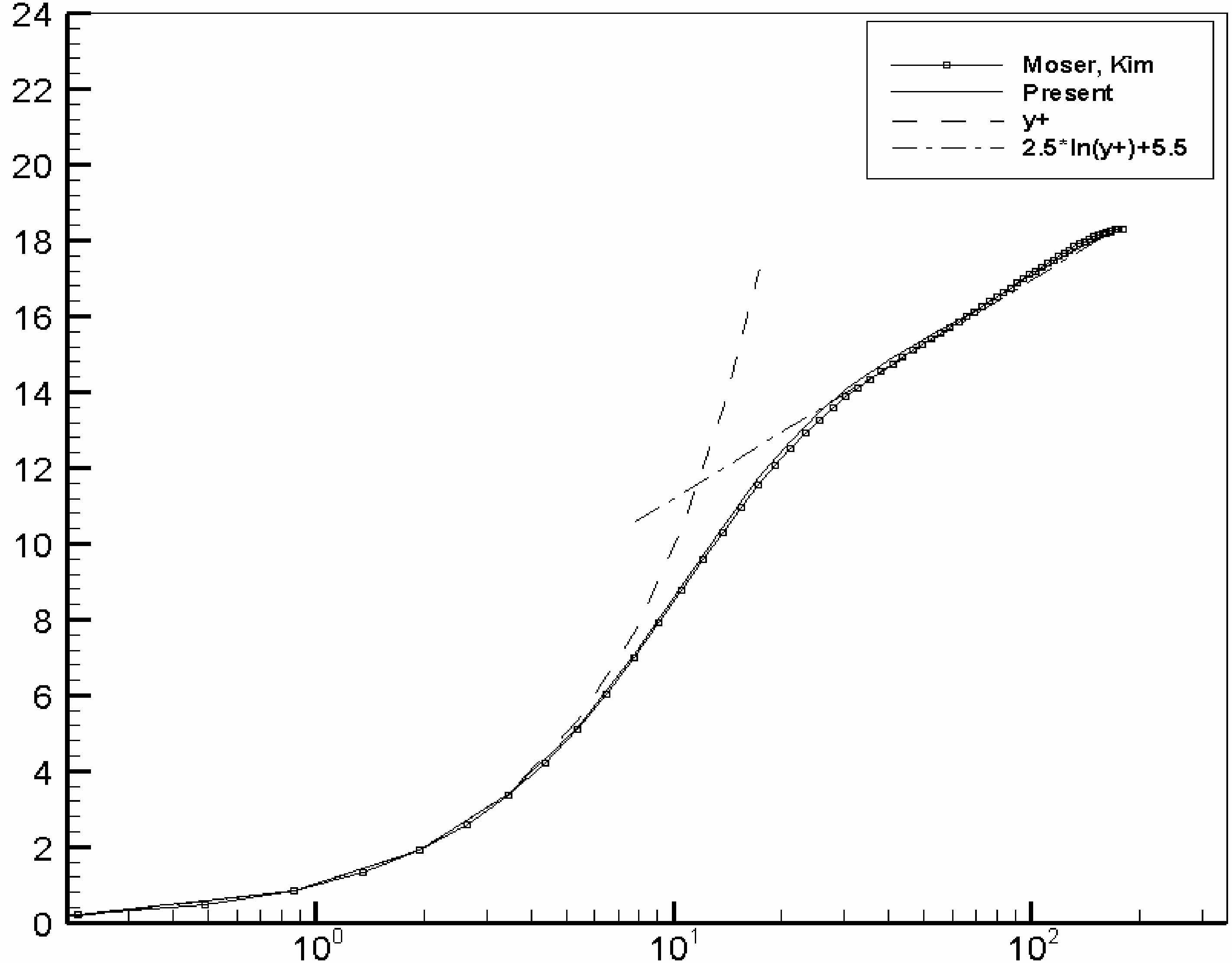
$$\boxed{\langle \omega^2_{k_x,k_y,k_z} \rangle > \lambda}$$

We test codes by adding a body force

$$\frac{\partial u}{\partial t} - \nu \Delta u + (u \cdot \nabla u) + \nabla p = f$$

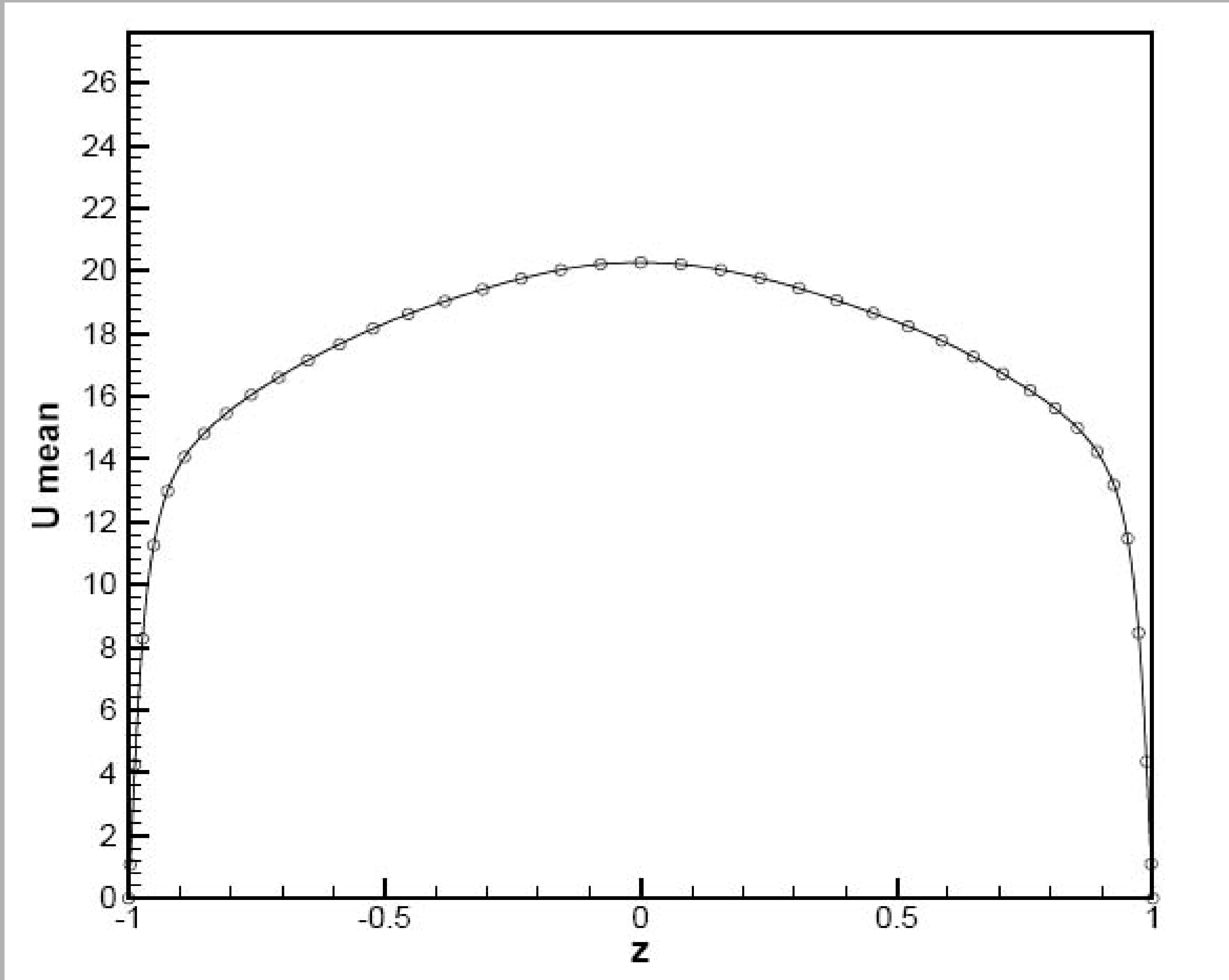
$$\nabla \cdot u = 0, \quad x \in \Omega$$

Standard comparisons were also made

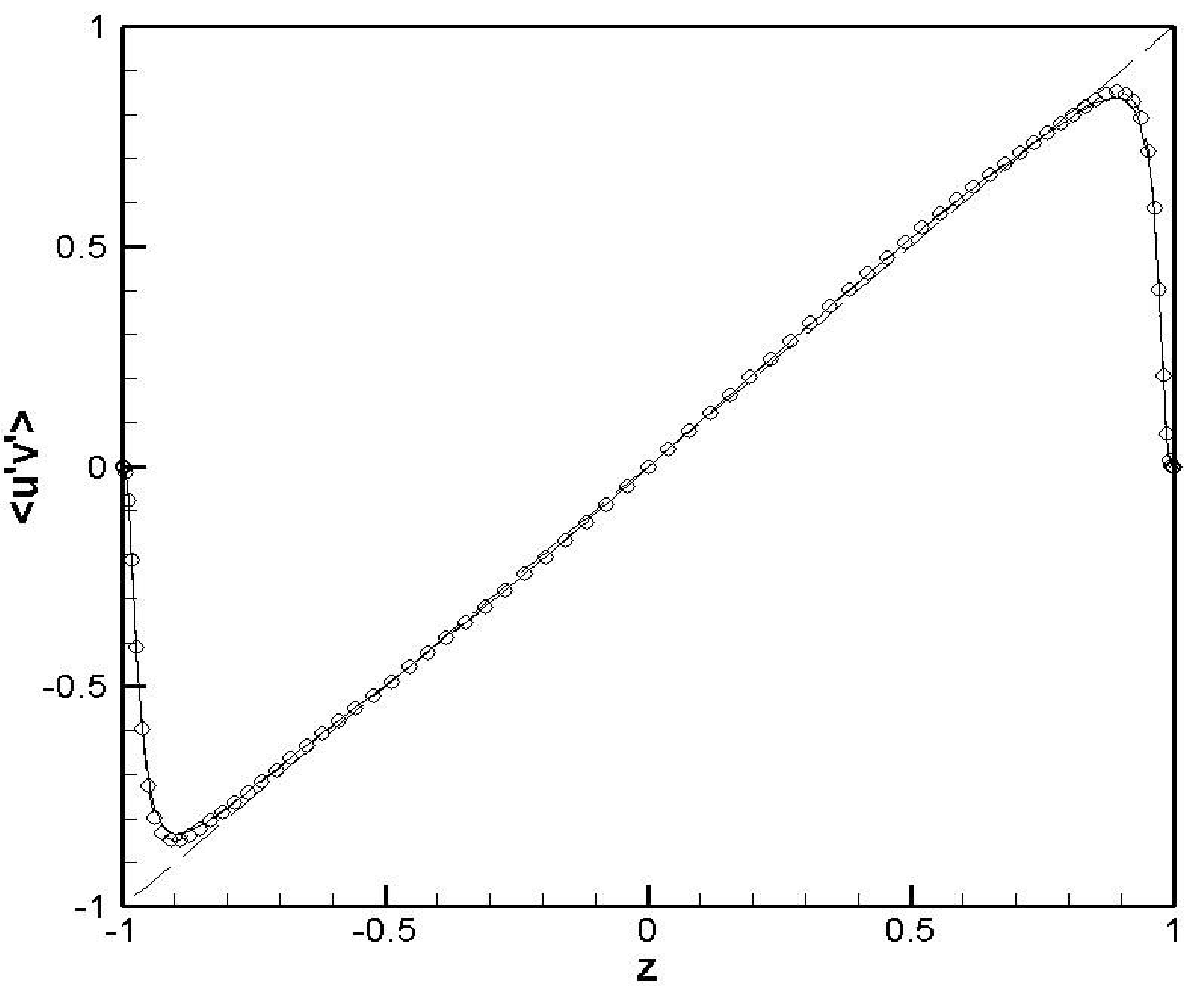


Near-wall velocity profiles

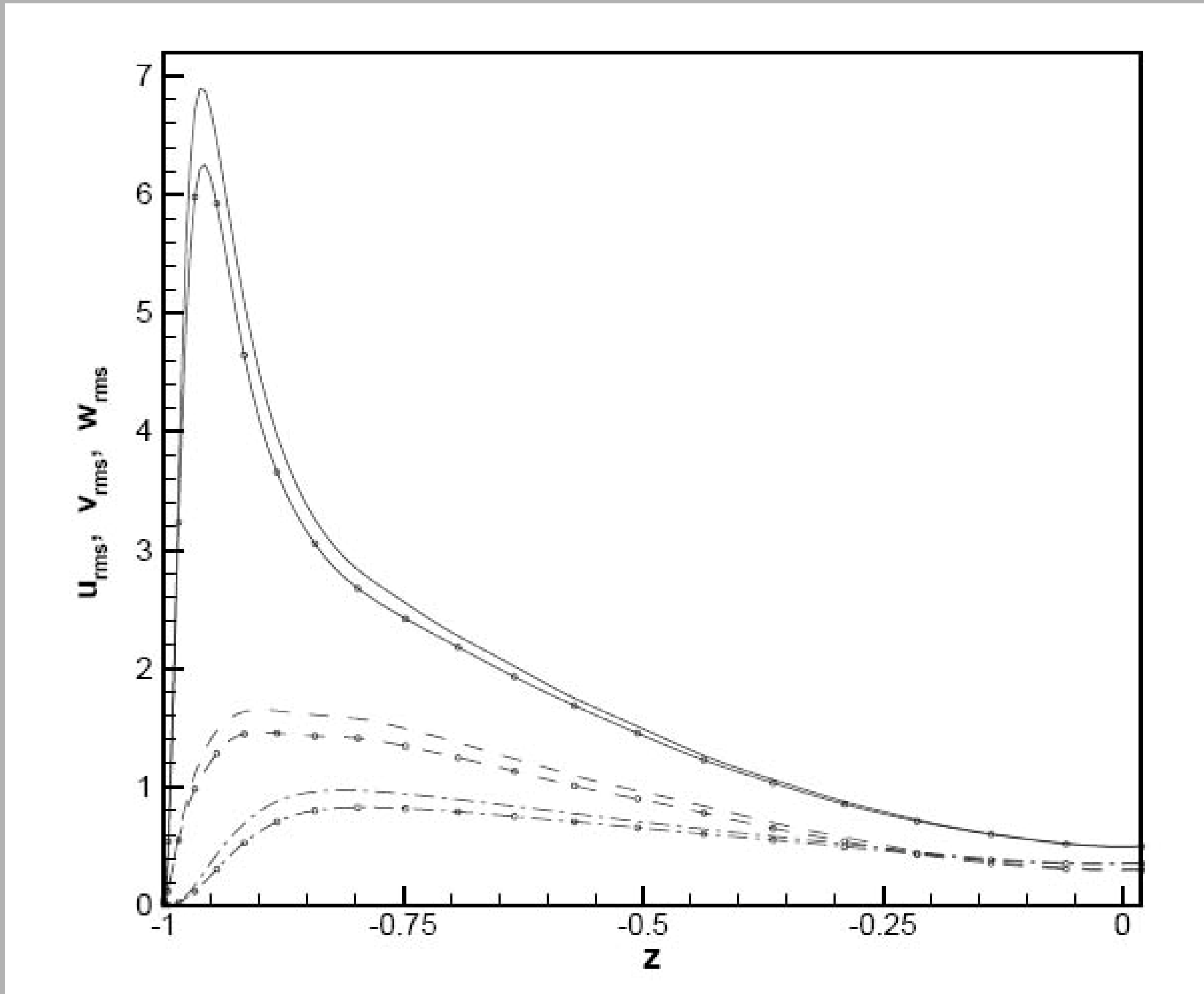
Mean velocity is reproduced closely



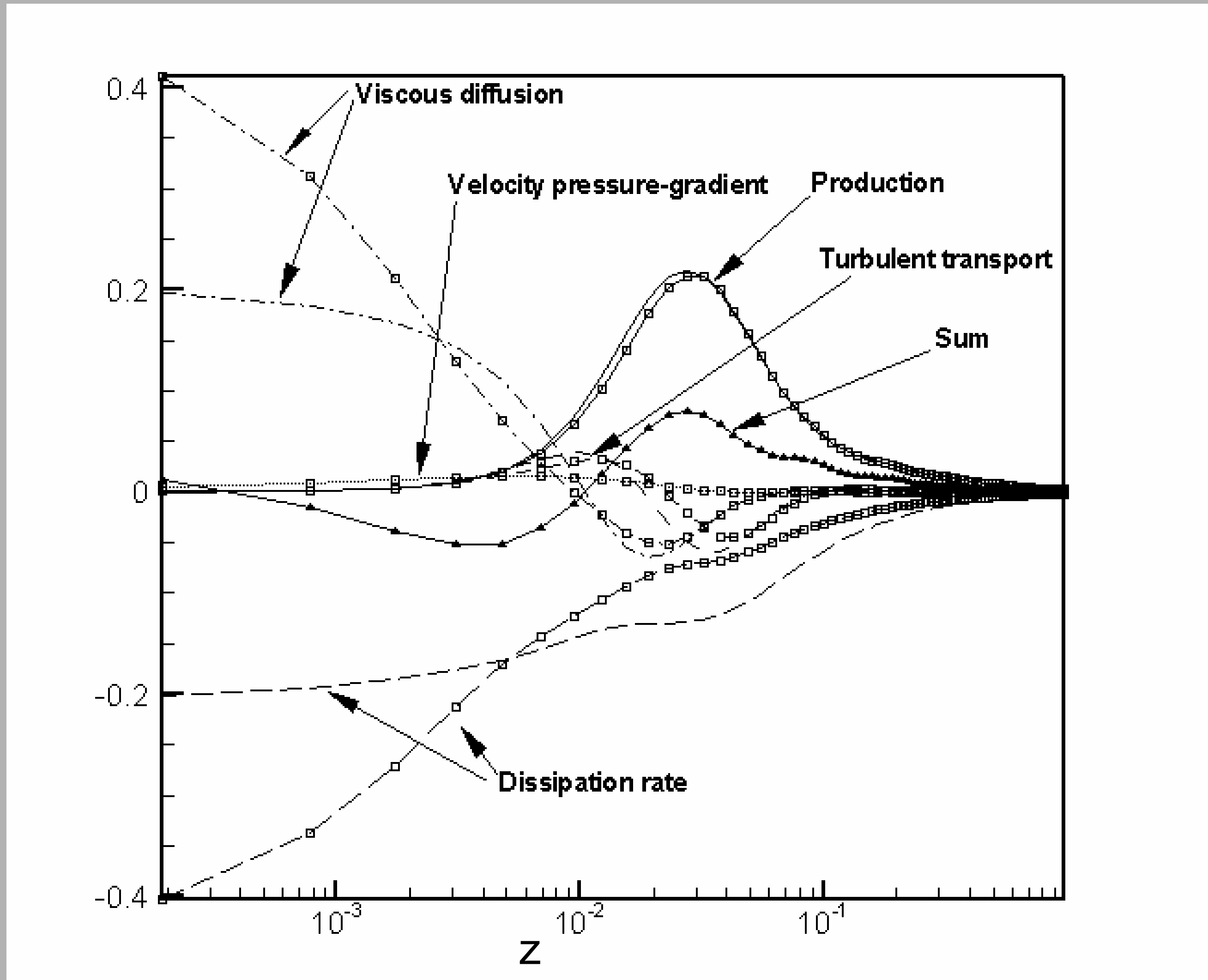
Shear stress is reproduced closely, too



Fluctuation intensity is reasonably good



Near the wall turbulence energy balance is not reproduced



Master-mode set size is close to the attractor dimension

$$N_{attr} \sim 10^{-6} L_x L_y \mathrm{Re}^{9/4}$$

$$\approx 20000$$

Re based on mean velocity and channel width