

ETC11, Porto, 25-28 June 2007

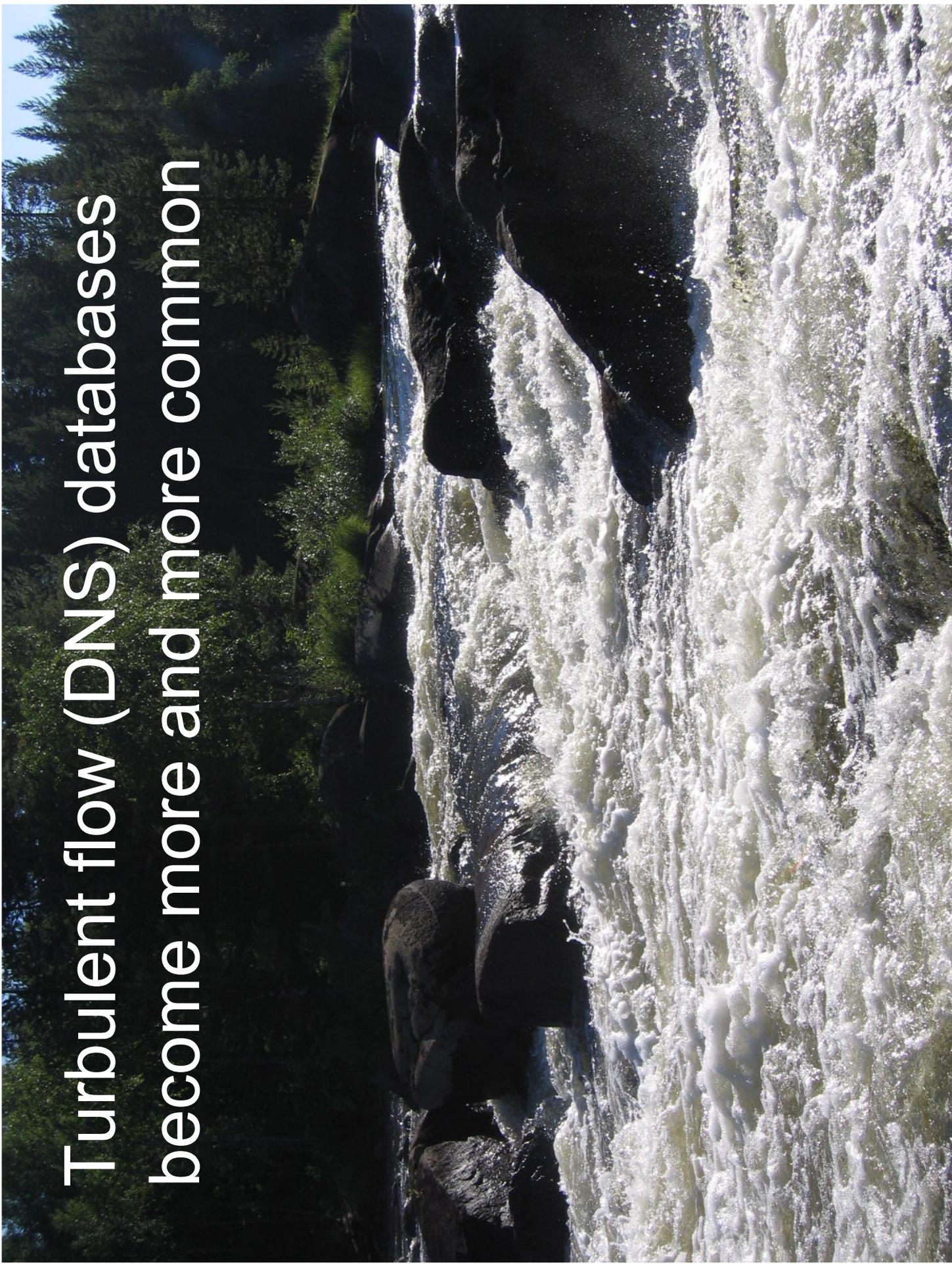
26.06. Room 3. Session 3.4 Wall-bounded flows (IV), 10:25-10:45.

Master-mode set for 3D turbulent channel flow

by Sergei Chernyshenko
and Maxym Bondarenko

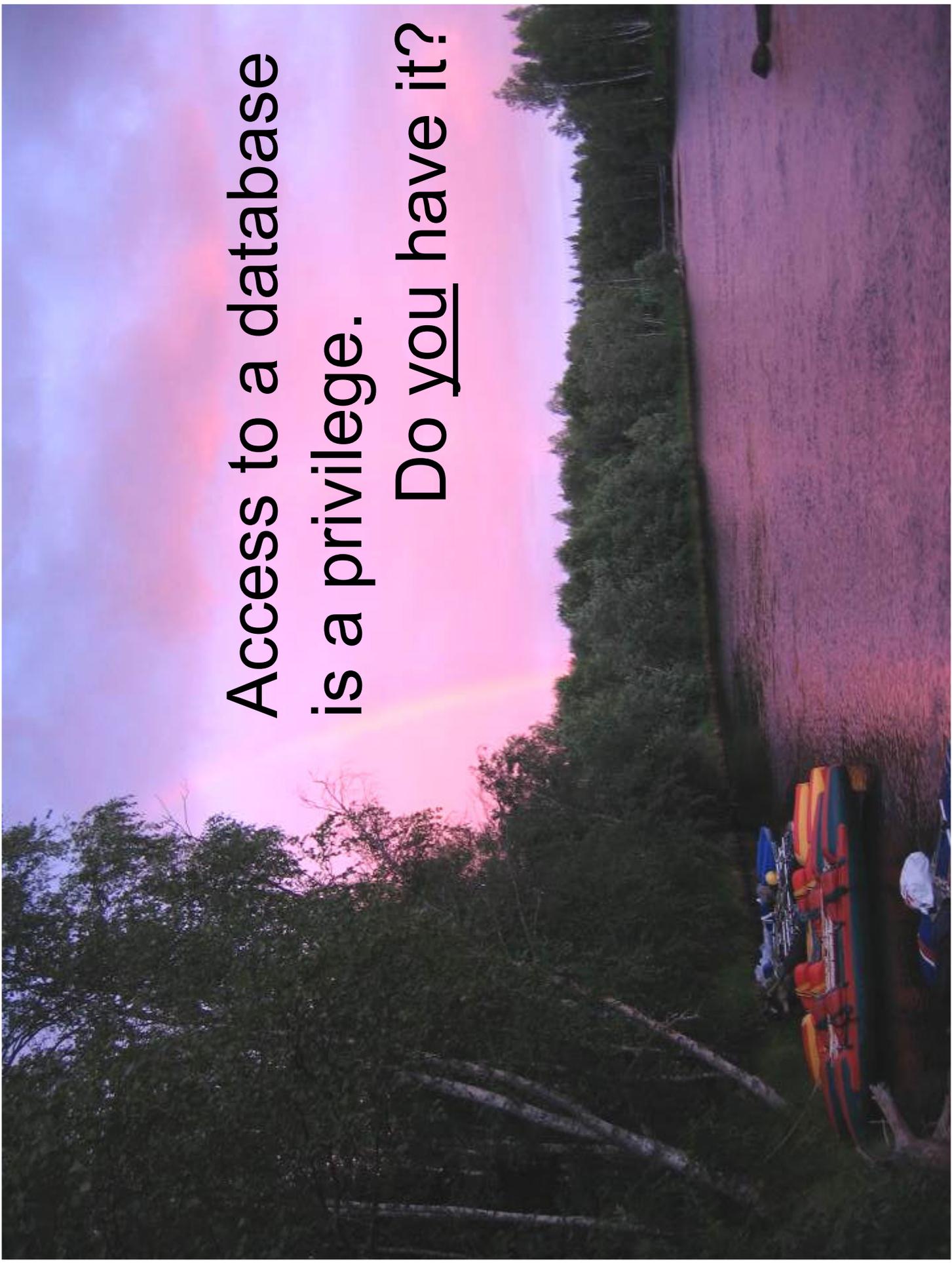
School of Engineering Sciences
University of Southampton
UK

Turbulent flow (DNS) databases
become more and more common



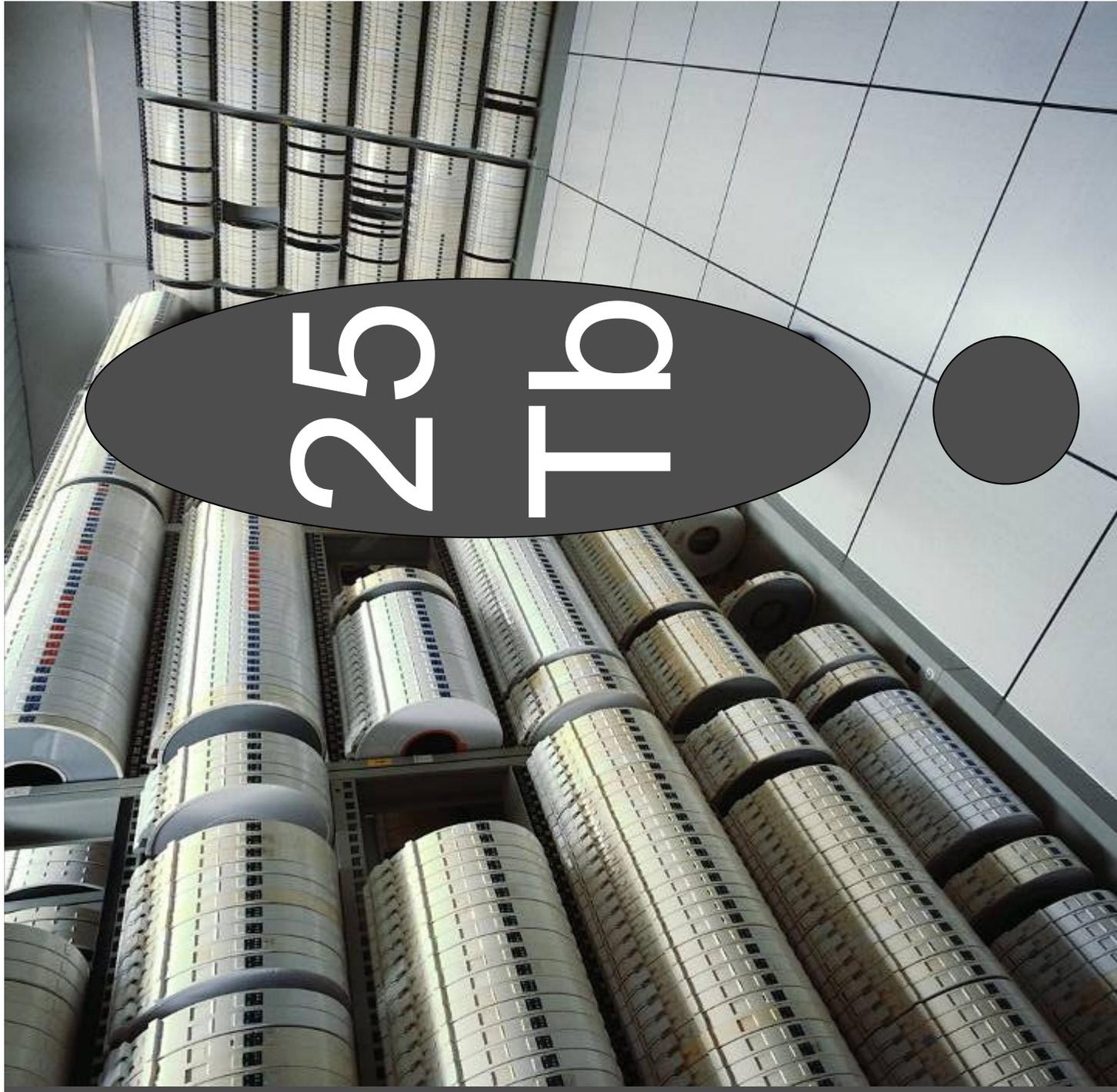
Access to a database
is a privilege.

Do you have it?



Full DNS
database
requires
too much
storage

25
Tb



Complete database can be small

CHANNEL FLOW DNS DATA

How to use:

1. Extract using *gzip* or *winzip*
 2. Rename *file.dat* to *input.dat*
 3. Run *readf*
 4. Check print out is the same as *check.dat*
-



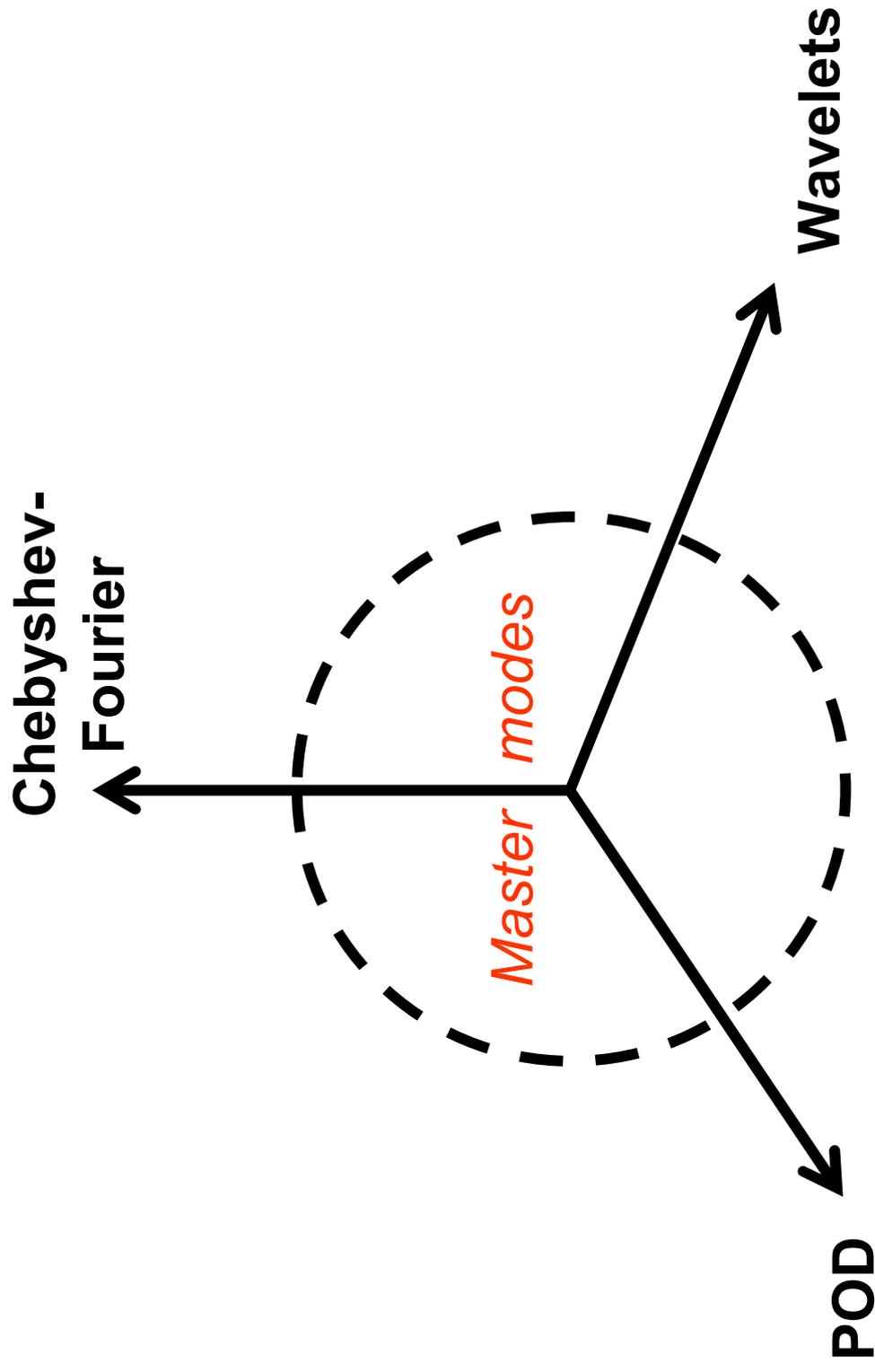
Aerodynamics and Flight Mechanics Research Group
SCHOOL OF ENGINEERING SCIENCES



Storing only the master-mode
set solves the problem

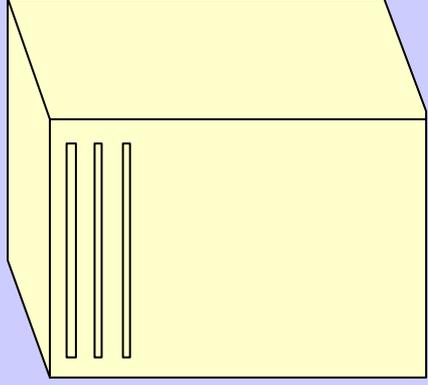


What master-modes are not

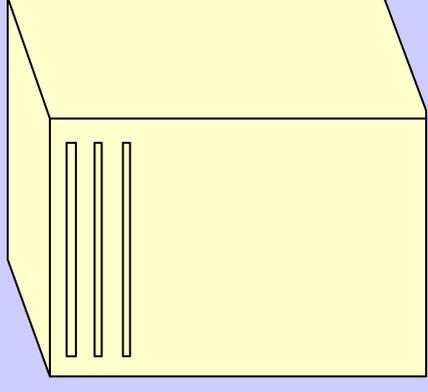


Master code dictates master-modes to the slave code

$$\mathbf{u}_i(\mathbf{x}) = \sum_{n=1}^S \hat{\mathbf{u}}_{in} \phi_n(\mathbf{x}), \quad t = t_i$$



$\hat{\mathbf{u}}_{i,m_1}, \dots, \hat{\mathbf{u}}_{i,m_K}$

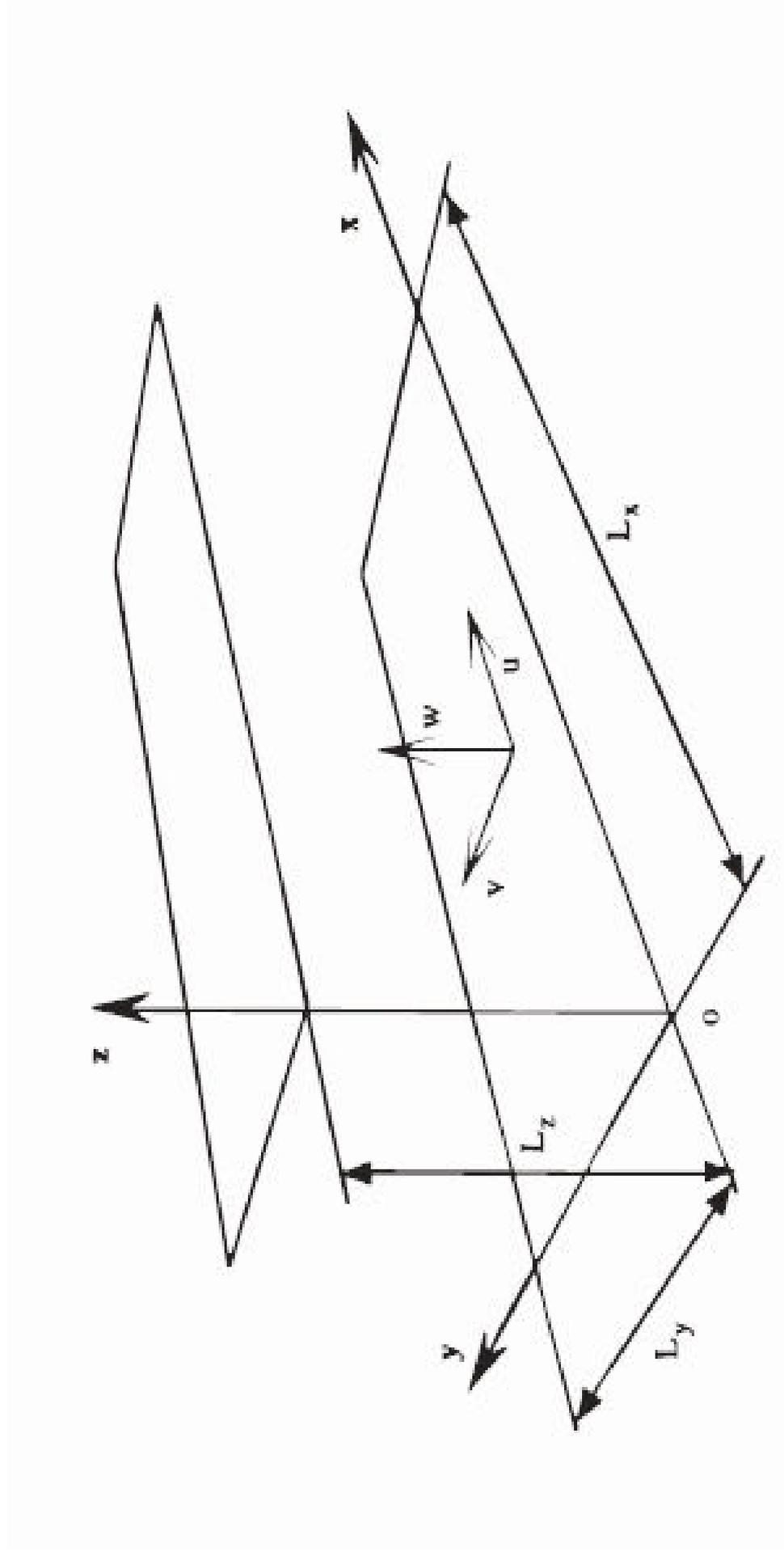


Master code

Slave code

$$\|\mathbf{u}_{\text{master}} - \mathbf{u}_{\text{slave}}\| \xrightarrow{?} 0$$

3D channel flow has a master-mode set

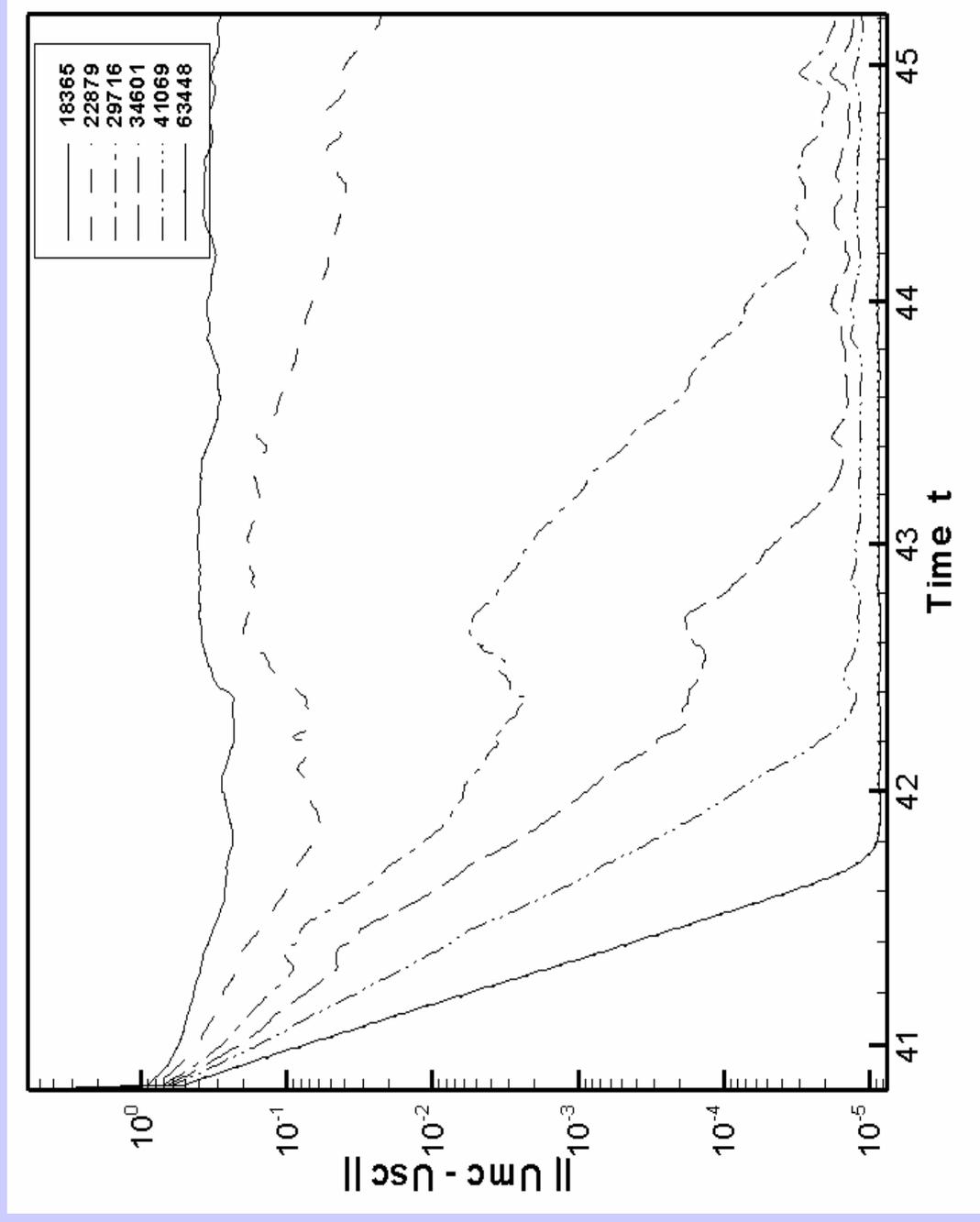


Our code works

- Sandham and Howard pseudo-spectral code is reliable
- We test codes by adding a body force
- Standard comparisons were also made

Slave solution converges to the master solution

$$Re_{\tau} = 360$$



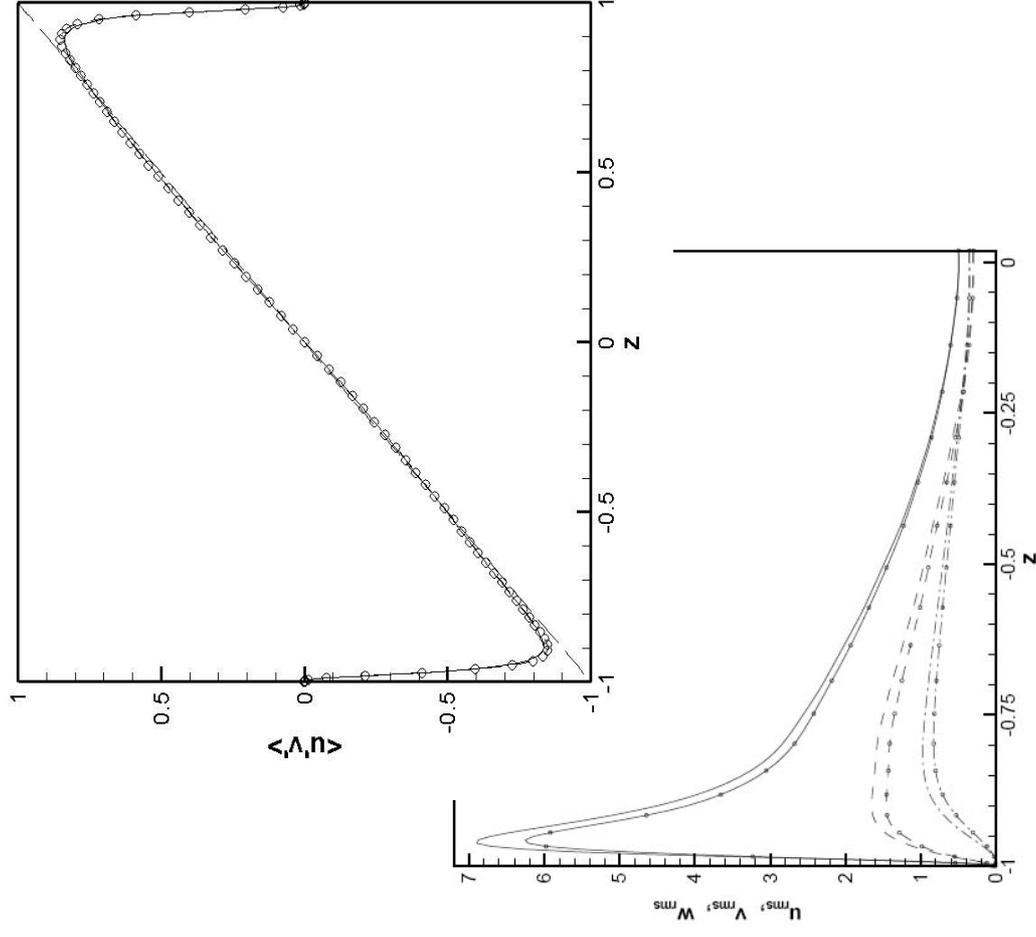
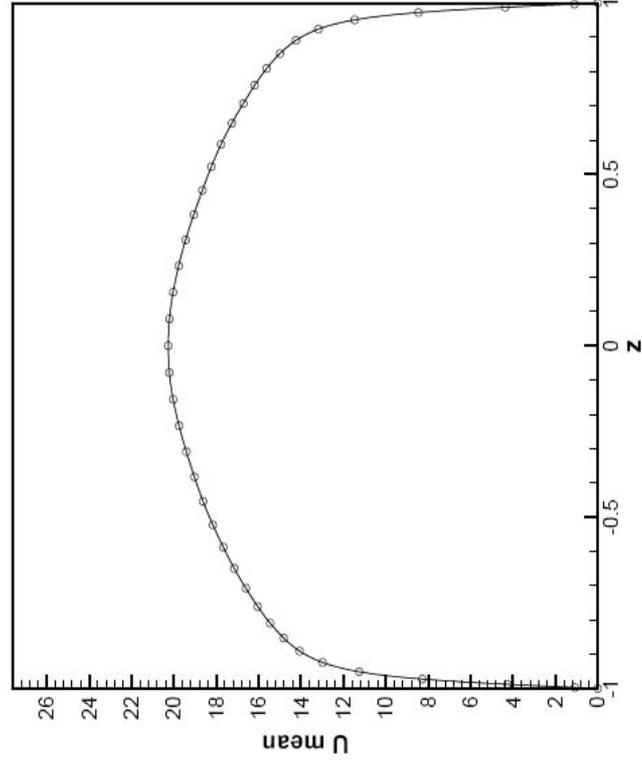
$6 \times 3 \times 2$ box

$K \sim 30000$

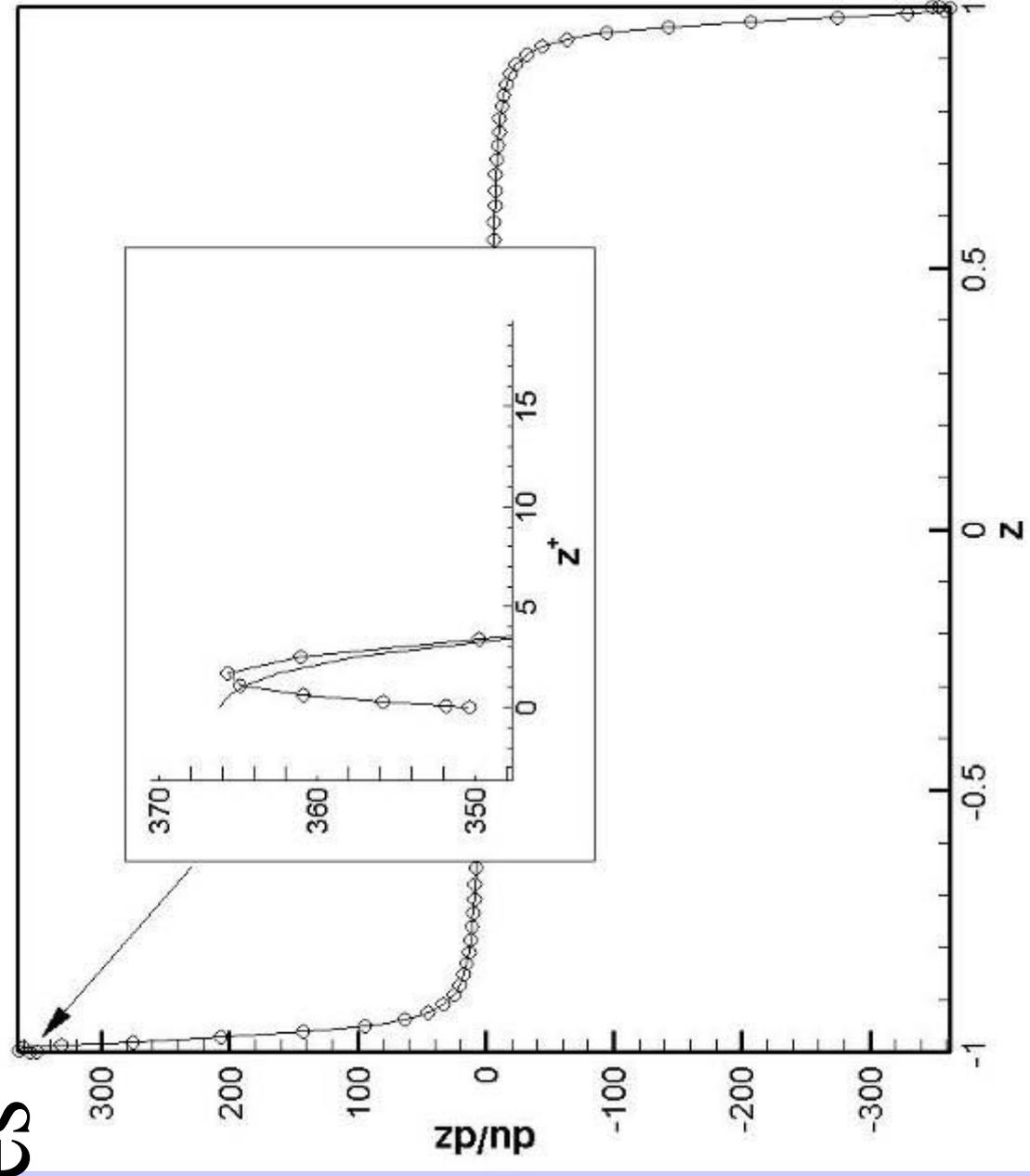
Master-mode-based database is 100 times smaller

- We had 2621440 modes in total
- Master-mode set size is less than 30000
- ~1%!

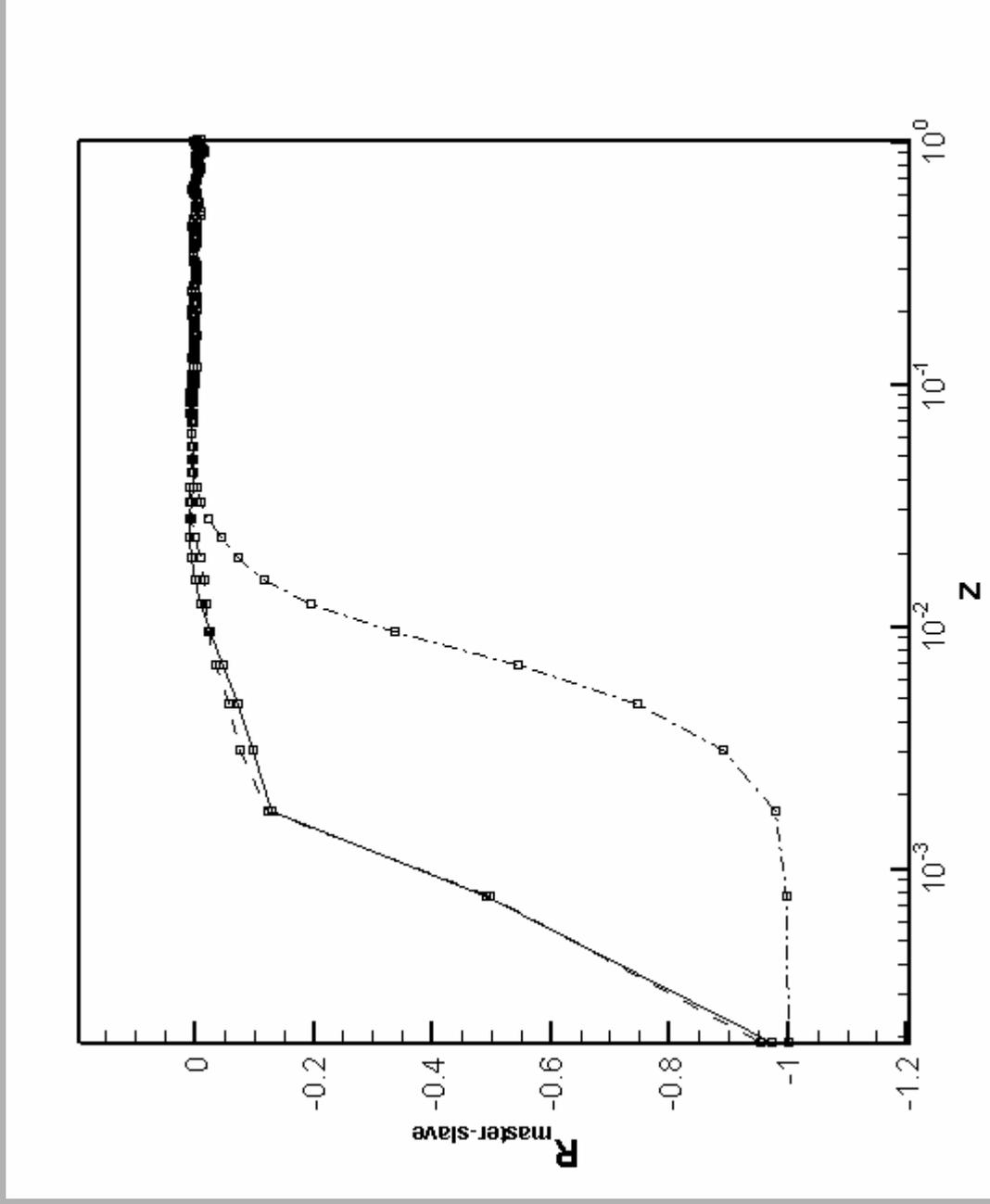
Master-modes alone provide a decent approximation



Near the wall there is a problem with derivatives

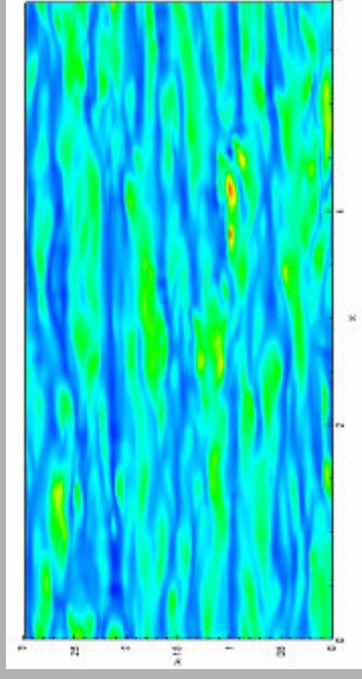


Near the wall master-slave correlation
is high

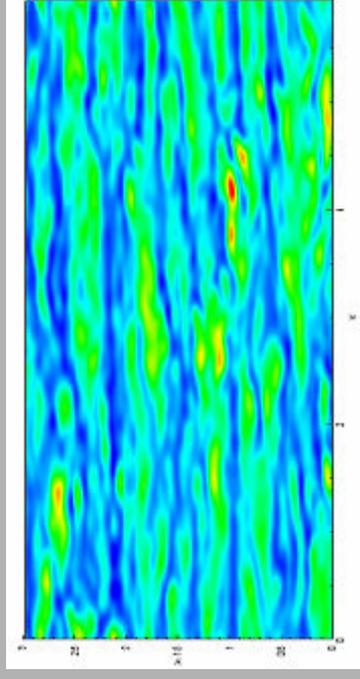


Streaks are in the master-mode set

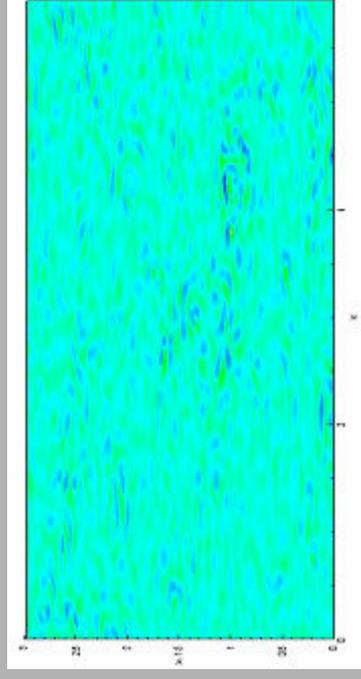
Full



Master

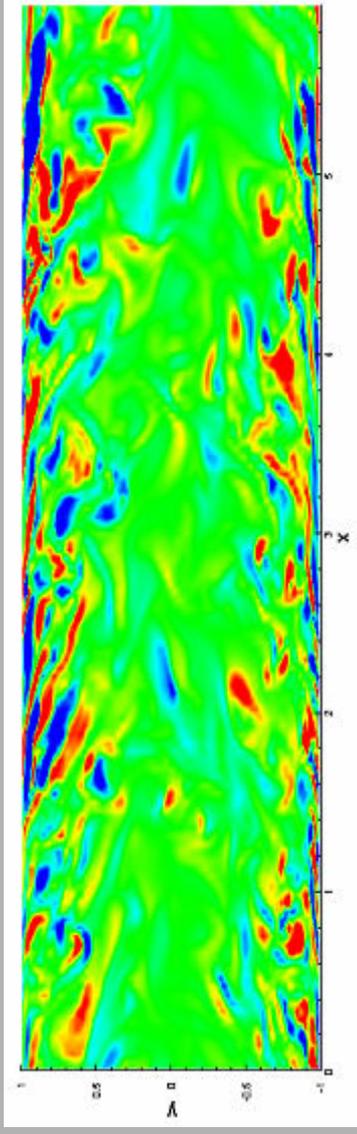


Slave

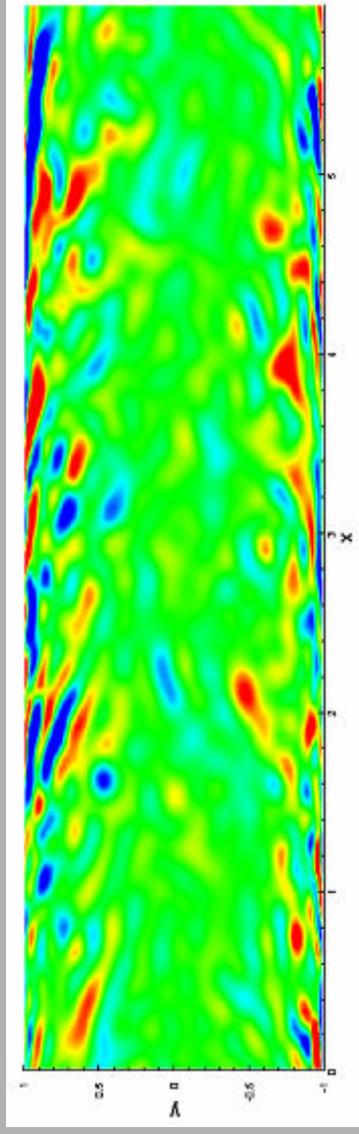


Vortices are mostly in master-mode set

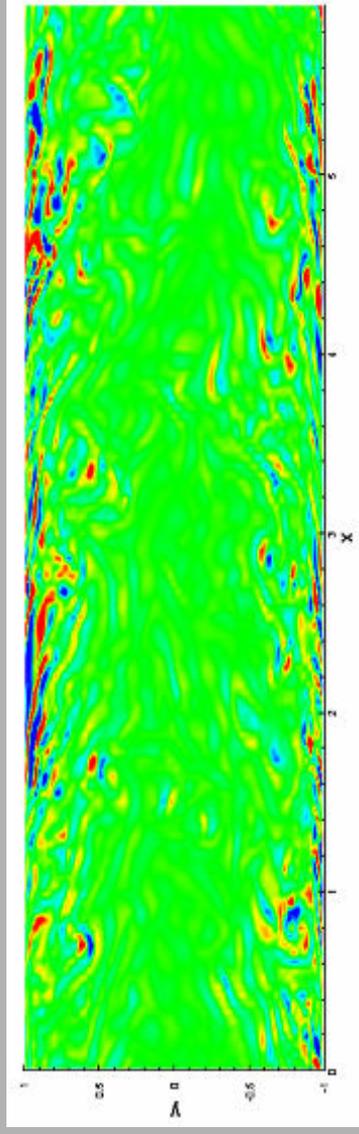
Full



Master



Slave

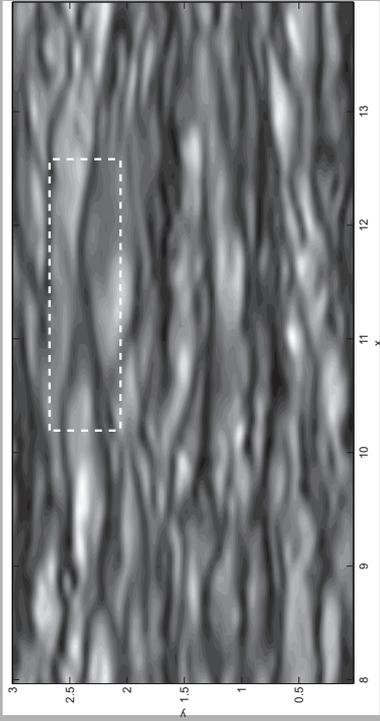


Master-mode database is best suited
for catching rare events

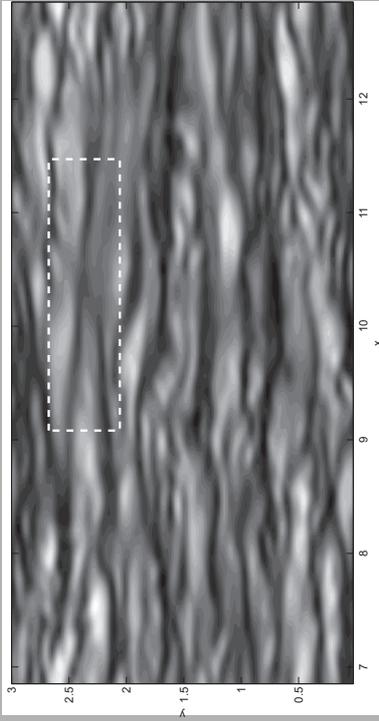
$$\mathbf{u}(x, y, z, t) = \mathbf{U}(x - ct, y, z)$$

$$\min_c \iint_V \sum_{i=1}^3 \left(\frac{\partial u_i}{\partial t} + c \frac{\partial u_i}{\partial x} \right)^2 \approx 0$$

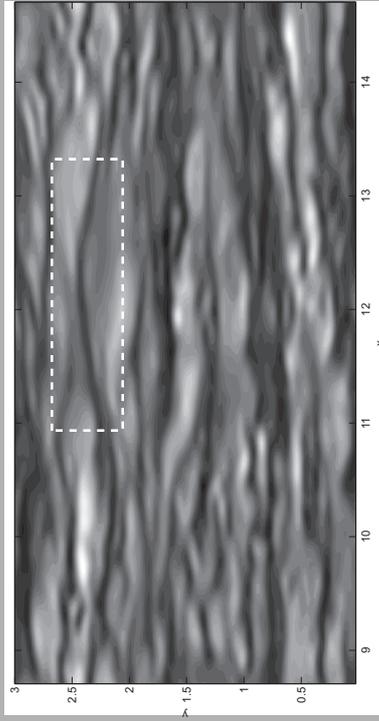
Travelling-wave-like object is detected



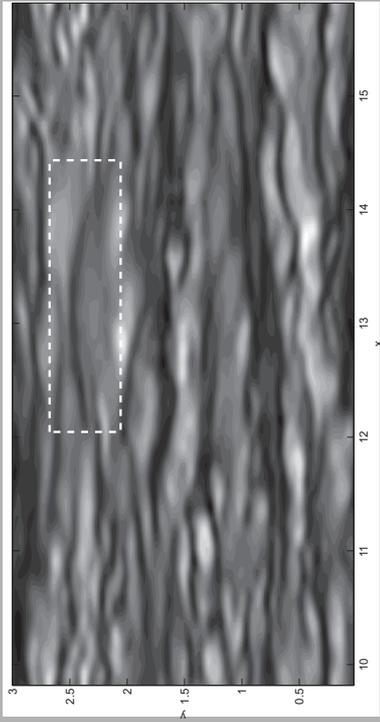
t_1



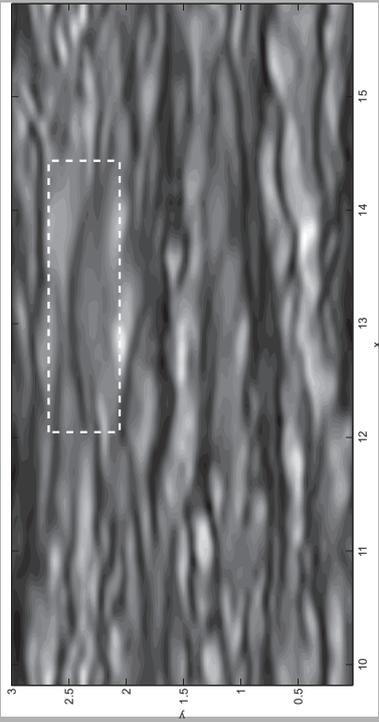
t_2



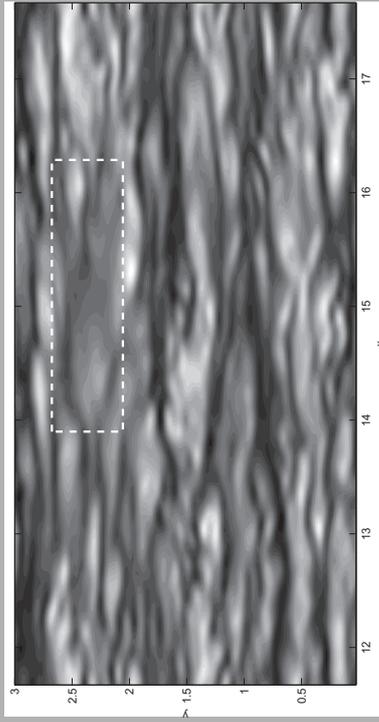
t_3



t_4



t_5



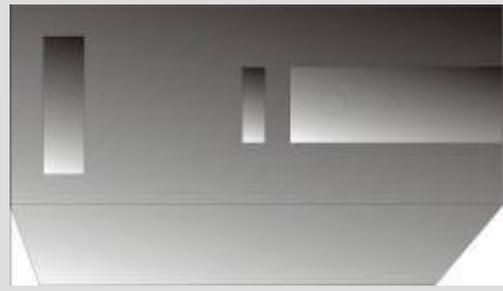
t_6

Can you have access to the database?
Right now, today?

One could use a full database but
they are too large!

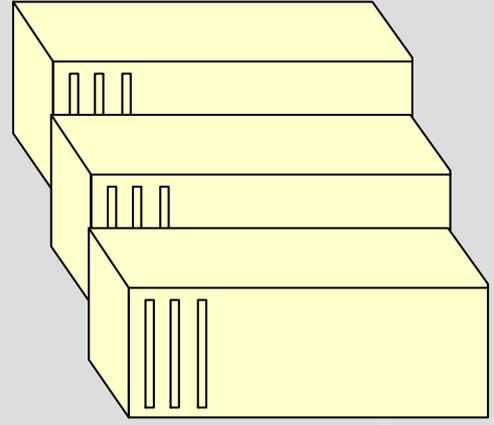


Storing only the master-mode set
solves the problem

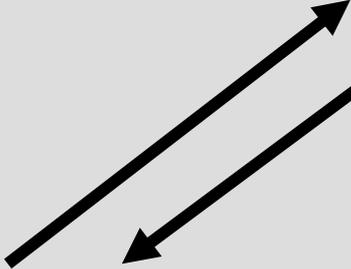
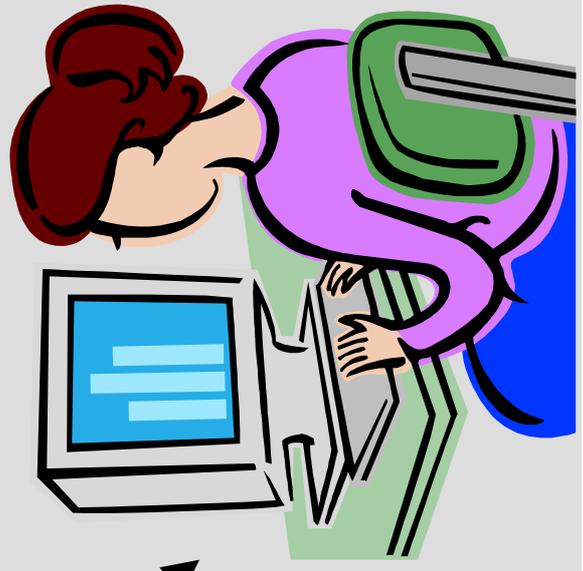


Web
server

Fortran
Compiler



Database



Master-mode database is online at www.dnsdata.afm.ses.soton.ac.uk

The screenshot shows a Microsoft Internet Explorer browser window. The title bar reads "DNS - Microsoft Internet Explorer". The address bar contains the URL "http://www.dnsdata.afm.ses.soton.ac.uk/". The browser interface includes a menu bar (File, Edit, View, Favorites, Tools, Help), a toolbar with navigation buttons (Back, Forward, Stop, Refresh, Home, Search, Favorites, Go), and a search bar. The main content area displays the website "DNS Database of Turbulent Flows". The page header includes the University of Southampton logo and a navigation menu with links: Home, Login, Profile, Cases, Applications, Web server, Help, Forum, Contact us, and a link to "Index.php". The main text of the page reads: "Welcome to the DNS database of turbulent flows at the Aerodynamics and Flight Mechanics (AFM) Research Group, School of Engineering Sciences, University of Southampton. Data on this server are provided free for academic usage by different contributors. Users are advised to check related paper for simulation details and numerical methods, and should cited listed references to give credit to the contributors." Below this, a paragraph states: "Apart from ascii files for turbulence statistics, binary files for full three-dimensional flowfields and other data have also been archived. The web-based service built on this server allows users manipulate those files with own code. Currently only FOTRAN90 code (.f90) can be uploaded and run." The status bar at the bottom shows the URL "http://www.dnsdata.afm.ses.soton.ac.uk/index.php" and the "Internet" icon.

DNS Database of Turbulent Flows

Welcome to the DNS database of turbulent flows at the Aerodynamics and Flight Mechanics (AFM) Research Group, School of Engineering Sciences, University of Southampton. Data on this server are provided free for academic usage by different contributors. Users are advised to check related paper for simulation details and numerical methods, and should cited listed references to give credit to the contributors.

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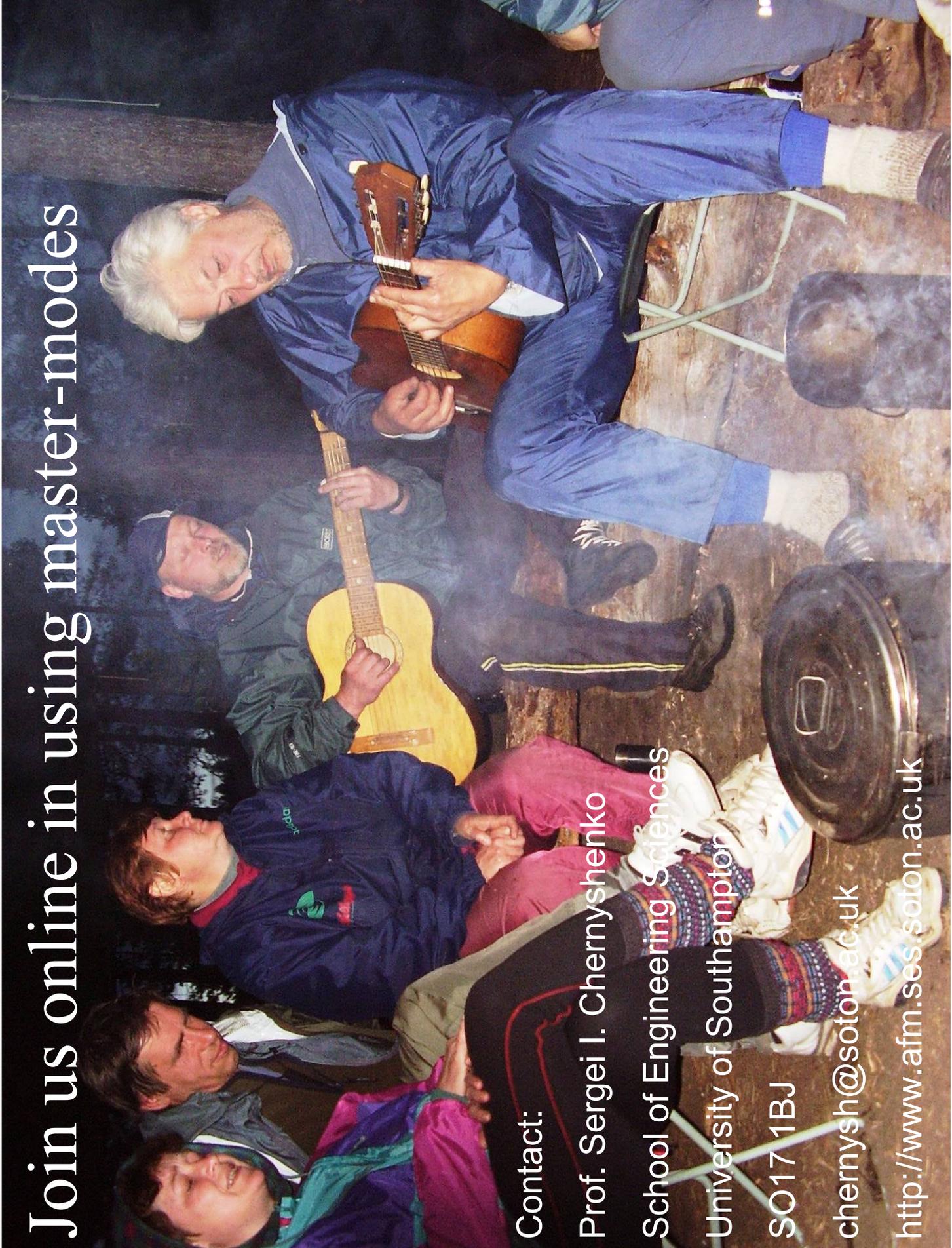
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Flight Mechanics Research Group

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<http://www.dnsdata.afm.ses.soton.ac.uk/index.php> Internet



Join us online in using master-modes

Contact:

Prof. Sergei I. Chernyshenko

School of Engineering Sciences

University of Southampton

SO17 1BJ

chernysh@soton.ac.uk

<http://www.afm.ses.soton.ac.uk>

Additional slides

Master-mode set determines the entire

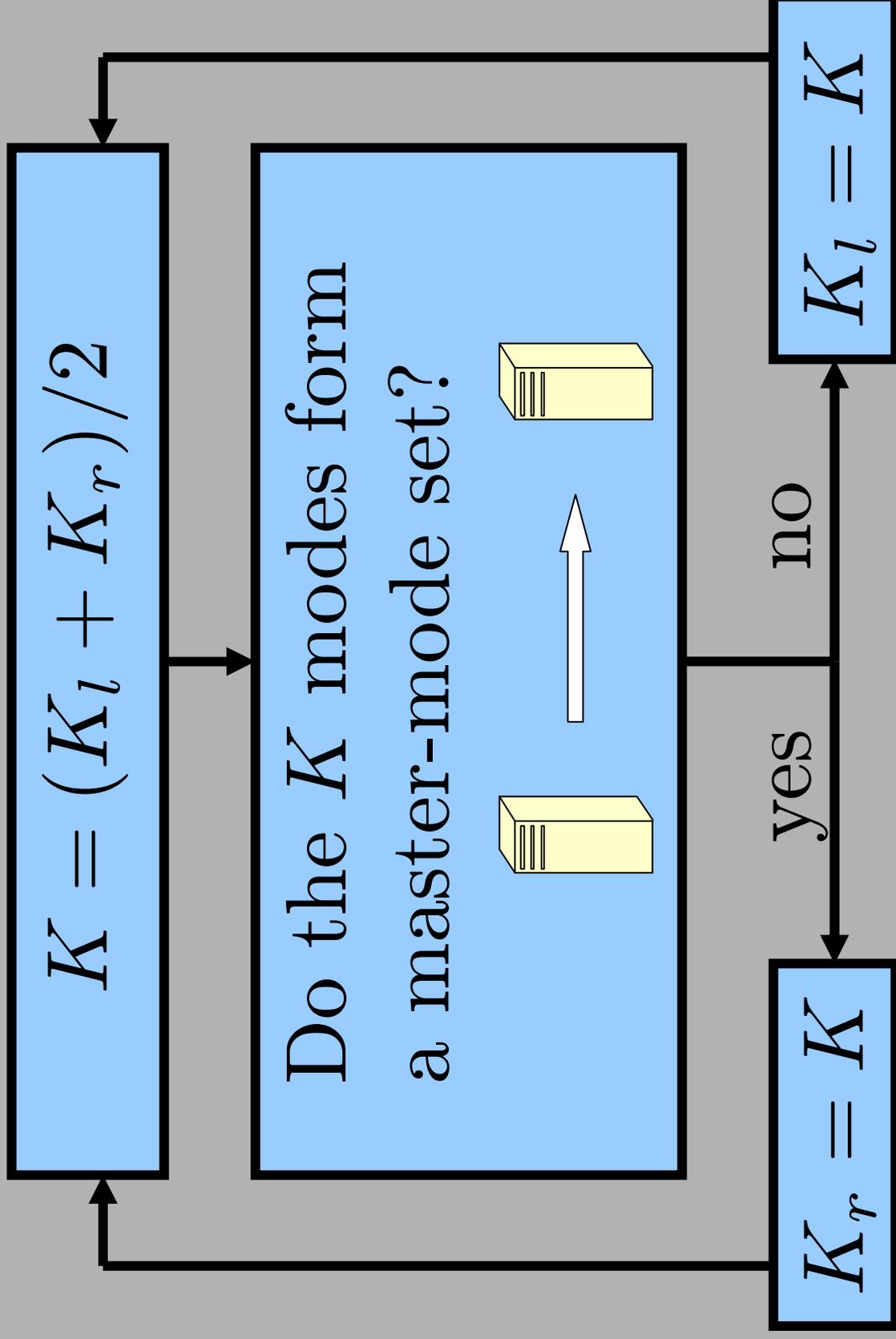
solution

$$\mathbf{u}_A = \sum_{n=1}^{\infty} A_n(t) \phi_n(\mathbf{x}), \quad \left| \begin{array}{l} A_n(t) = B_n(t), \\ 1 \leq n \leq N \end{array} \right.$$

\Downarrow

$$\|\mathbf{u}_A - \mathbf{u}_B\| \rightarrow 0 \quad \text{as } t \rightarrow \infty.$$

Dichotomy is faster than trial-and-error



Ordering by mean amplitude is best

$$Re_{\tau} = 180, L_x \times L_y \times L_z = 4 \times 3 \times 2$$

Method	
Wavenumber	$K > 5000$
Energy-based	$2800 < K < 3500$
Enstrophy-based	$K \sim 4800$

Time marching is done by applying a recurrent formula to mode amplitudes

$$\mathbf{u}_i(\mathbf{x}) = \sum_{n=1}^S \hat{\mathbf{u}}_{in} \phi_n(\mathbf{x}), \quad t = t_i$$

$$\hat{\mathbf{u}}_{i+1,n} = D_n(\hat{\mathbf{u}}_{i,1}, \hat{\mathbf{u}}_{i,2}, \dots, \hat{\mathbf{u}}_{i,S})$$

Master-mode set definition

$$\mathbf{v}_i(\mathbf{x}) = \sum_{n=1}^S \hat{\mathbf{v}}_{i,n} \phi_n(\mathbf{x}) \quad M = \{m_1, \dots, m_K\}$$

$$\hat{\mathbf{v}}_{i+1,n} = \begin{cases} \hat{\mathbf{u}}_{i+1,n} & , \quad n \in M \\ D_n(\hat{\mathbf{v}}_{i,1}, \dots, \hat{\mathbf{v}}_{i,K}) & , \quad n \notin M \end{cases}$$

$$\|\mathbf{v}_i(\mathbf{x}) - \mathbf{u}_i(\mathbf{x})\| \rightarrow 0 \text{ as } i \rightarrow \infty \quad \forall \mathbf{v}_1(\mathbf{x})$$

$\Leftrightarrow M$ is a master-mode set

Olson and Titi (2003) ordered modes
by wavenumbers

$$\mathbf{u} = \sum_{k_x, k_y} \hat{\mathbf{u}}(t, k_x, k_y) e^{i(k_x x + k_y y)}$$

$$k_x^2 + k_y^2 < \lambda$$

Ordering by mean amplitude is similar
to ordering by energy

$$\mathbf{u} = \sum_{k_x, k_y, k_z} \hat{\mathbf{u}}_{k_x, k_y, k_z}(t) e^{i(k_x x + k_y y)} T_{k_z}(z)$$

$$\langle u_{k_x, k_y, k_z}^2 \rangle > \lambda$$

Can enstrophy-based ordering be better?

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}$$

$$\boldsymbol{\omega} = \sum_{k_x, k_y, k_z} \hat{\boldsymbol{\omega}}_{k_x, k_y, k_z}(t) e^{i(k_x x + k_y y)} T_{k_z}(z)$$

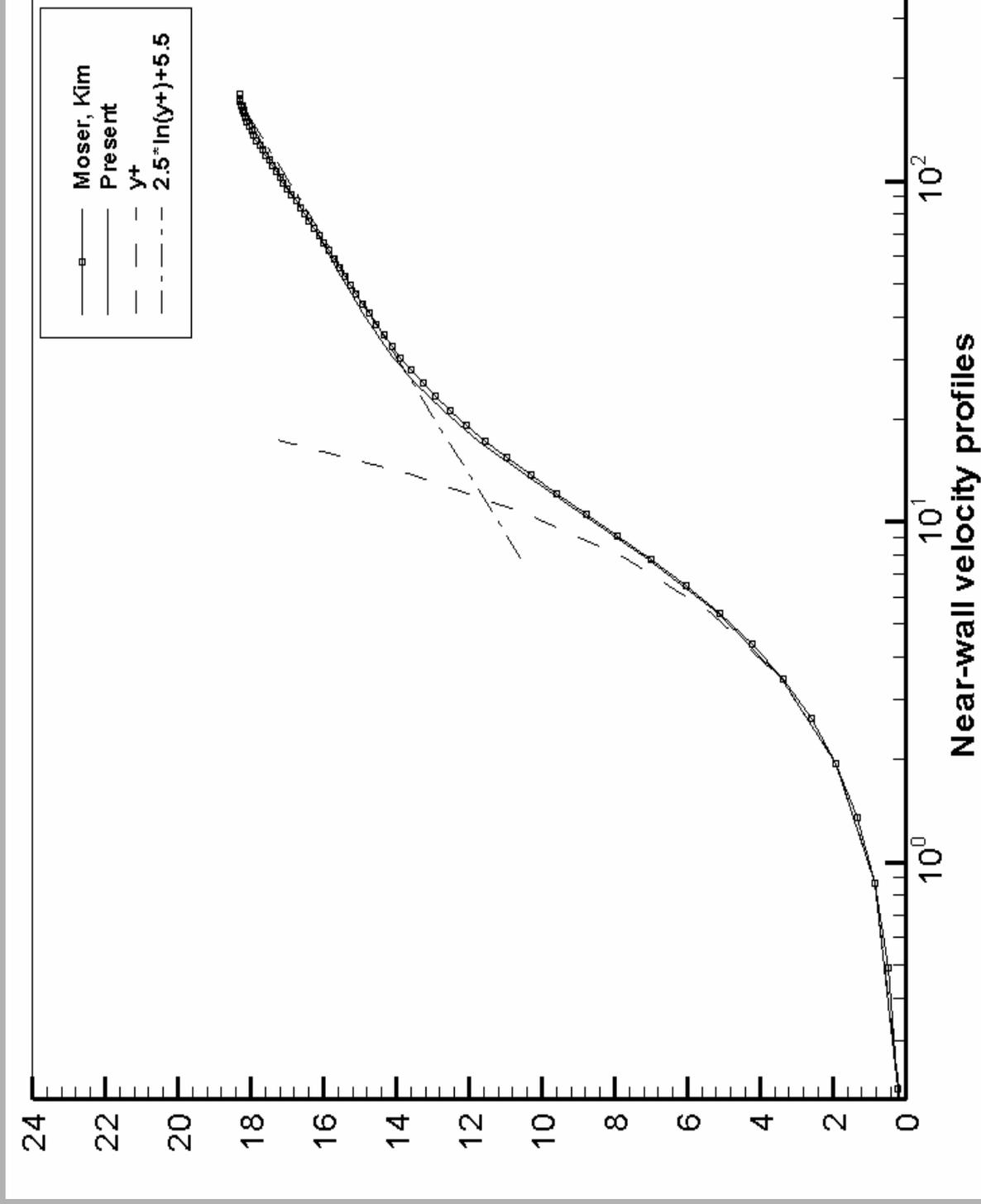
$$\langle \omega_{k_x, k_y, k_z}^2 \rangle > \lambda$$

We test codes by adding a body force

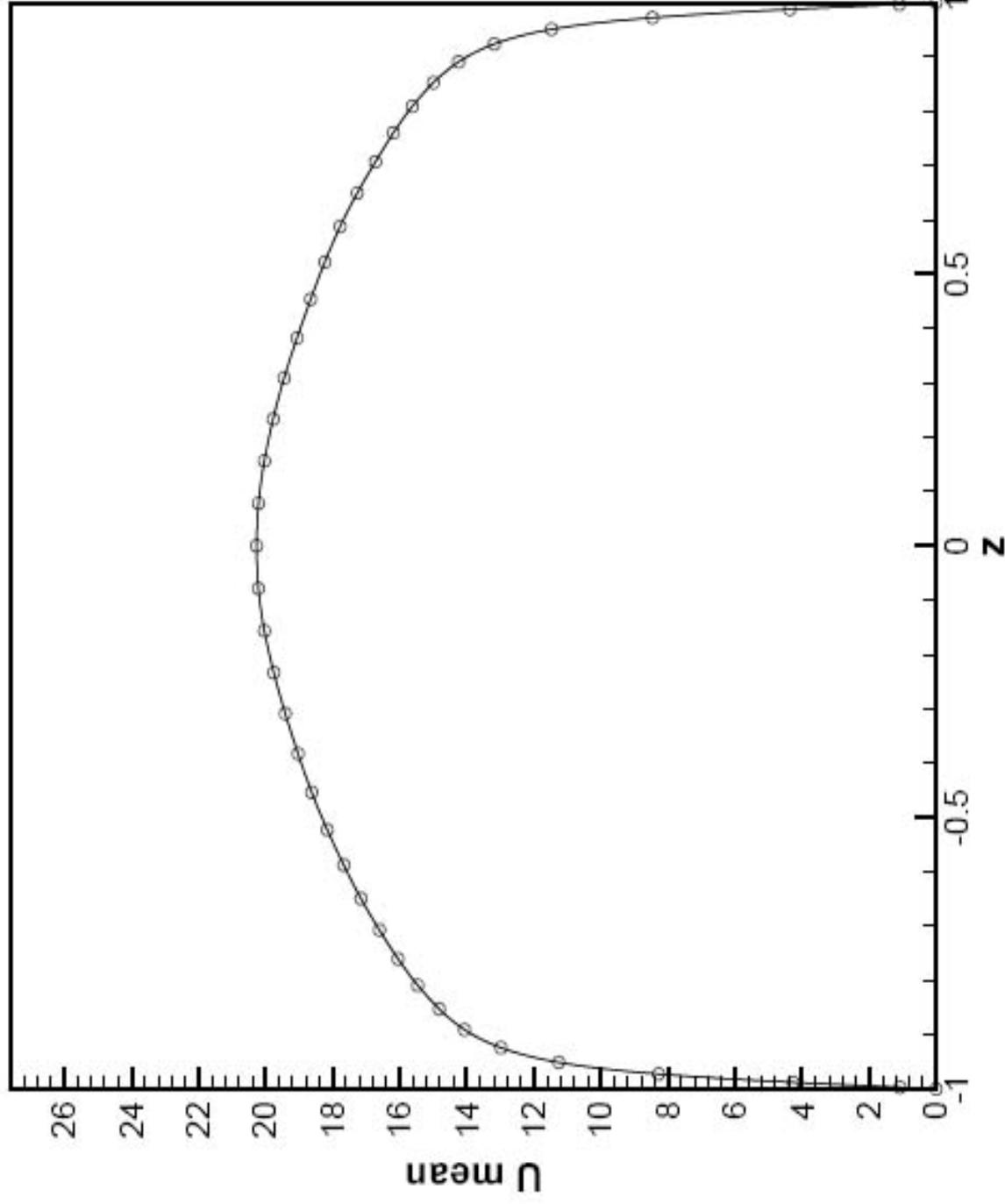
$$\frac{\partial u}{\partial t} - \nu \Delta u + (u \cdot \nabla u) + \nabla p = f$$

$$\nabla \cdot u = 0, \quad x \in \Omega$$

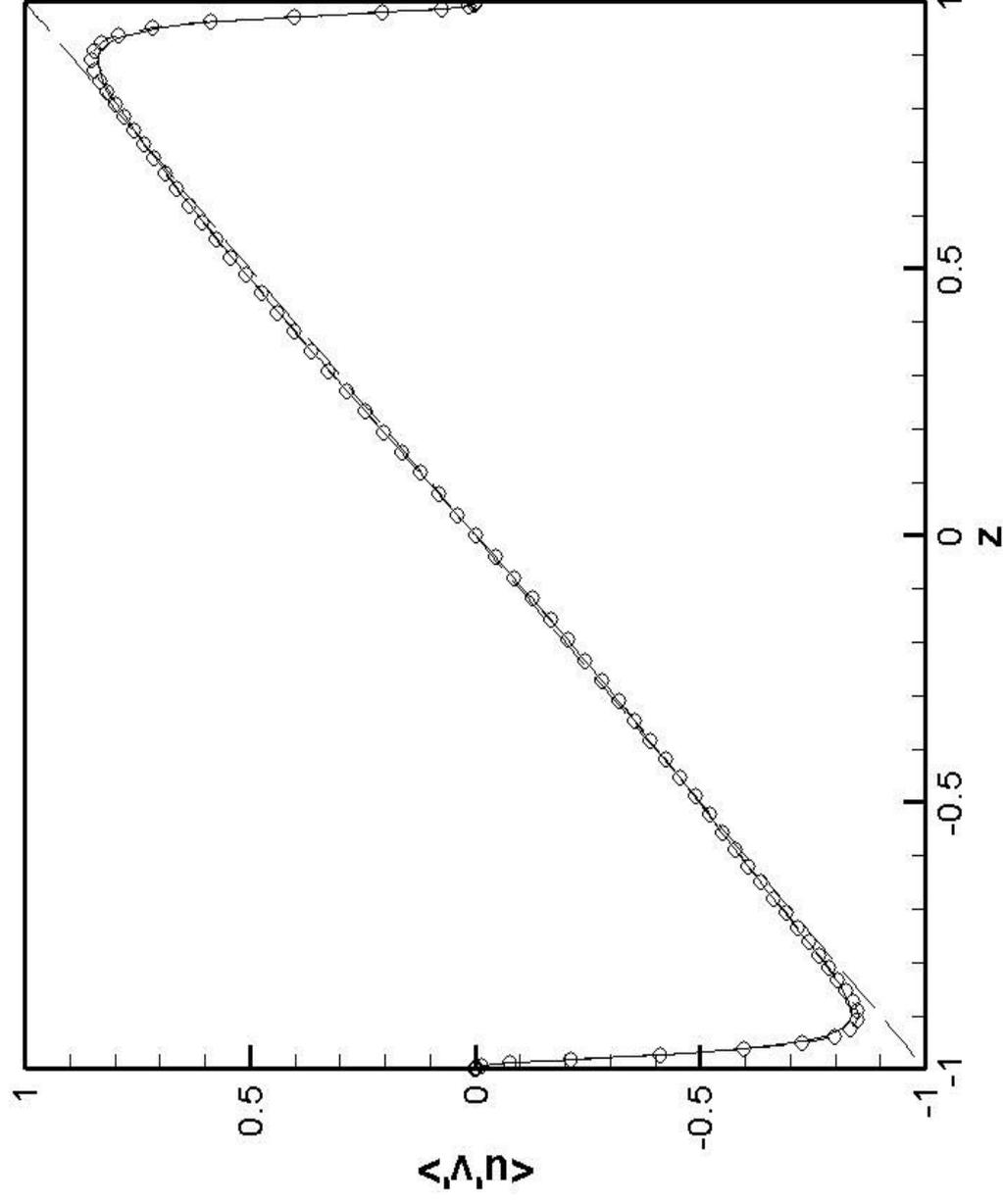
Standard comparisons were also made



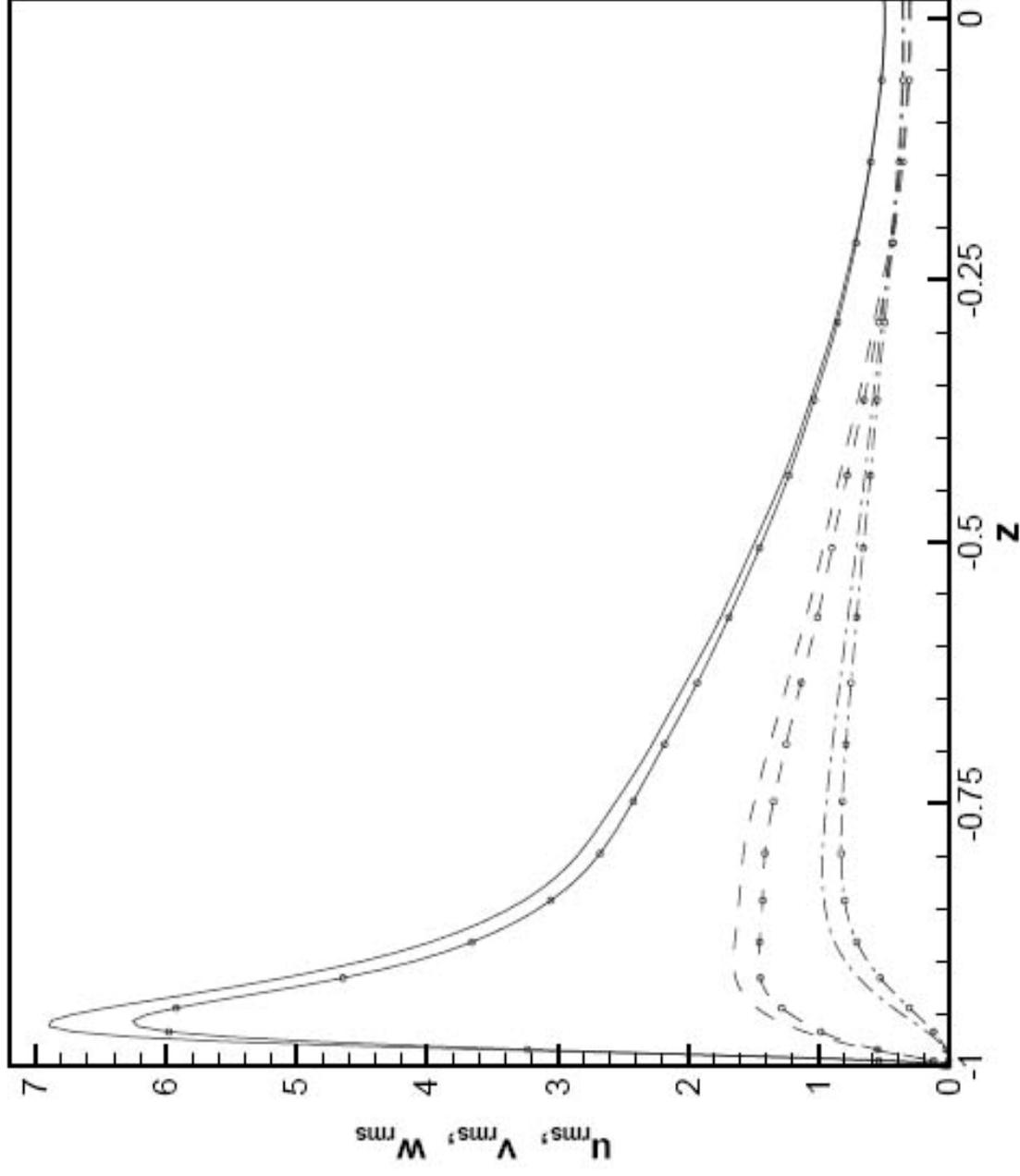
Mean velocity is reproduced closely



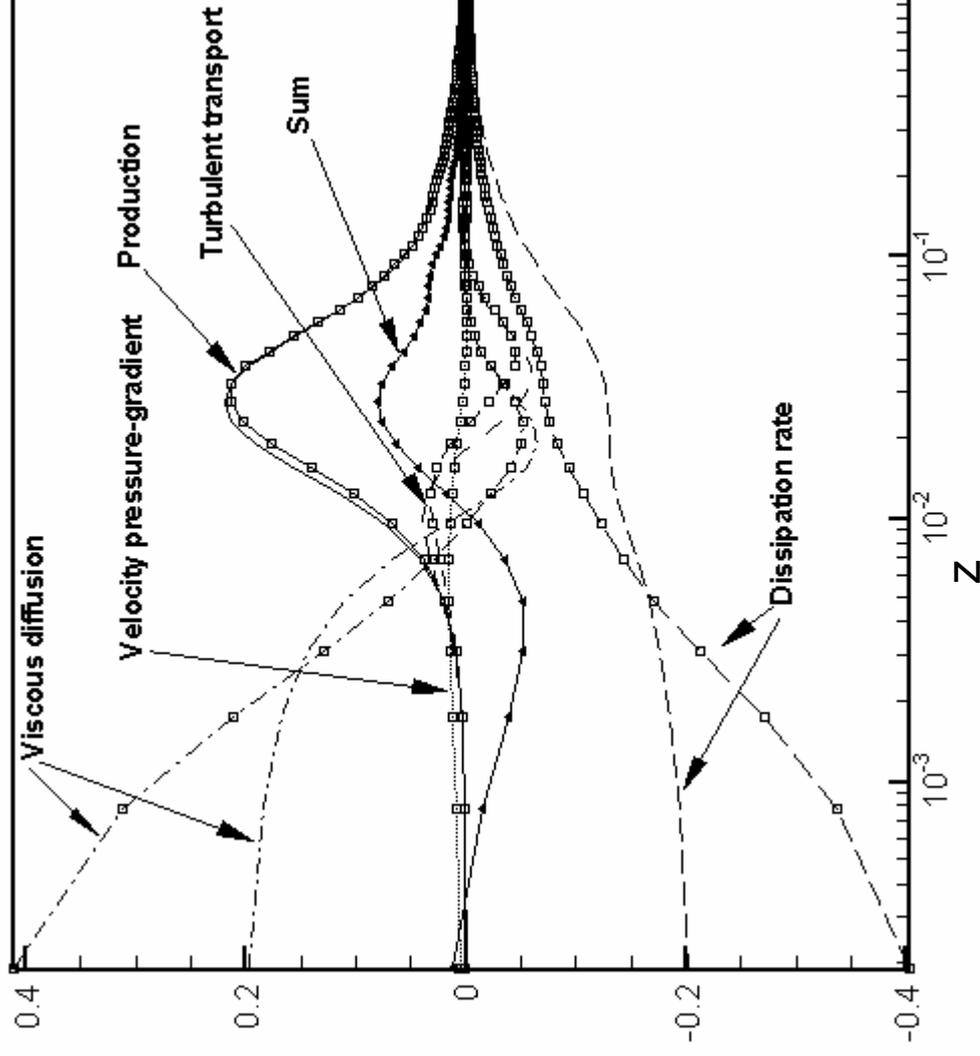
Shear stress is reproduced closely, too



Fluctuation intensity is reasonably good



Near the wall turbulence energy balance is not reproduced



Master-mode set size is close to the attractor dimension

$$N_{attr} \sim 10^{-6} L_x L_y \text{Re}^{9/4} \\ \approx 20000$$

Re based on mean velocity and channel width