Direct Numerical Simulation of Structural Vacillation in the transition to Geostrophic Turbulence

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Outline

- ## Convection in the rotating annulus

  - ### Typical progression of flow types toward turbulence

- ## The numerical model

- ## Results

  - ### Overview over results

  - ### A case of weak Structural Vacillation (?)

  - ### Flow field analysis and spectral analysis
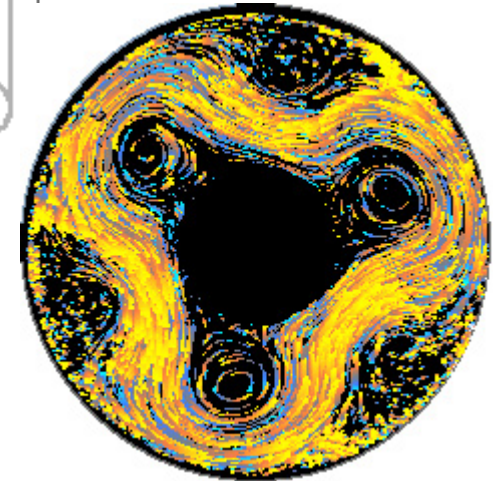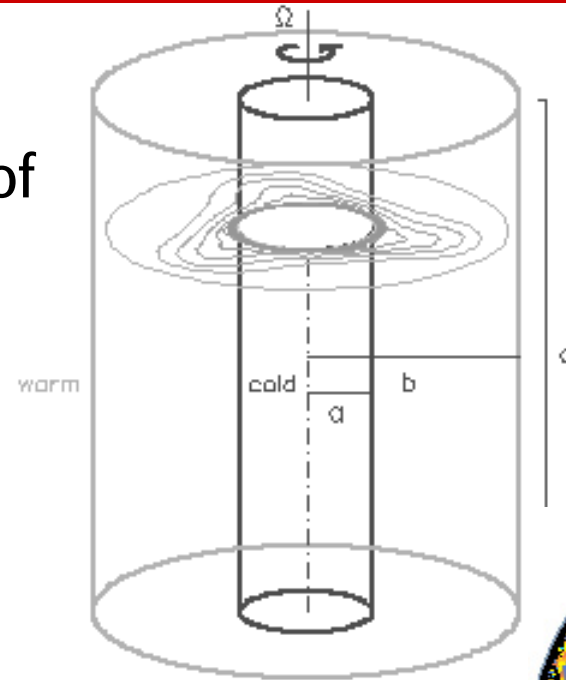
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Rotating Annulus

Annular convection chamber of
   height d and gap width $b - a$

Mounted on a turntable
   rotating at Ω

Filled with a fluid of
   – Prandtl number Pr,
   – kinematic viscosity ν,
   – density ρ,
   – volume expansion coefficient α

Outer wall heated and inner wall cooled
   temperature difference ΔT

Process and flow types

- Moderate rotation:

 baroclinic 'sloping' convection

- Fast rotation:

 centrifugal convection

- Bifurcation sequences for both

 – azimuthal basic flow

 – Regular, steady wave pattern, drifting through annular channel

 – Time-dependent flow ('Vacillation')

 (periodic, quasi-periodic, chaotic)

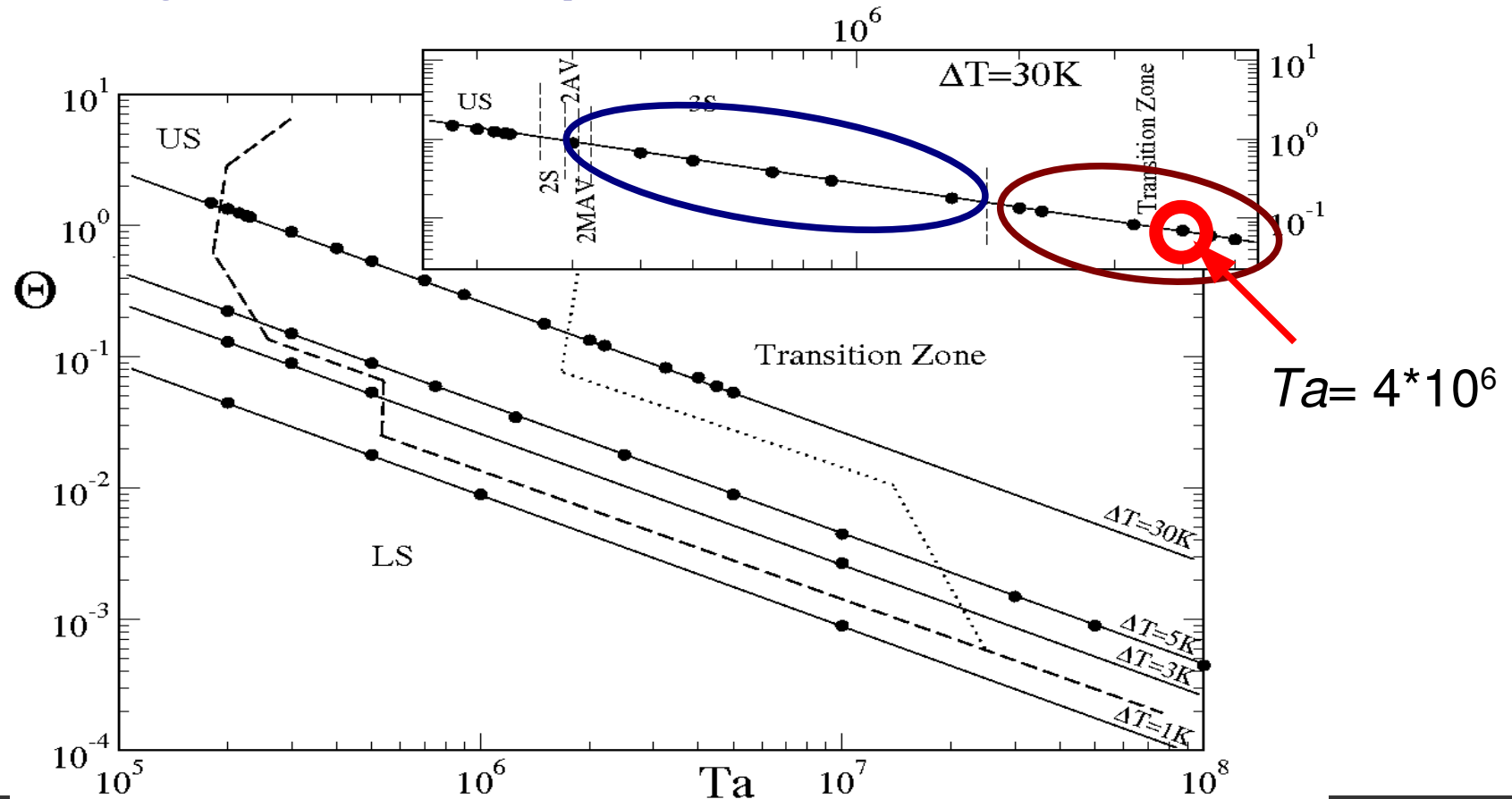 – Irregular fluctuations ('Structural Vacillation')

 – Geostrophic turbulence

The system

- $a = 34.8$mm, $b = 60.2$mm, ($L = 25.6$mm), $d = 100$mm with flat and rigid upper and lower boundaries

- Temperature difference $\Delta T = 30$K

- Rotation rate $\Omega = 52$rad/s (~9Hz, 500rpm)

- Filled with air:

  $\nu = 1.7e^{-5}$ m^2/s; $\kappa = 2.4e^{-5}$ m^2/s; ($Pr = 0.707$); $\alpha = 1/293$ K^{-1}

- $Ra = 40379$ or $Ra_\omega = 284{,}787$

- $Ta = 4 \times 10^6$

The numerical model

- Navier-Stokes and energy equations for a Boussinesq fluid

- Spectral model
  - Chebyshev Polynomials for radius *(r):* 109

    and axial direction *(z):* 109

    at Gauss-Lobatto collocation points
  - Fourier modes in the azimuthal direction: 256

- Time stepping: Adams-Bashforth/Backward Differentiation

- Integrations carried through for several drift periods of the dominant mode

Regime diagram in *Ta - Θ plane*

$Ta = 4*10^6$

DNS of Structural Vacillation
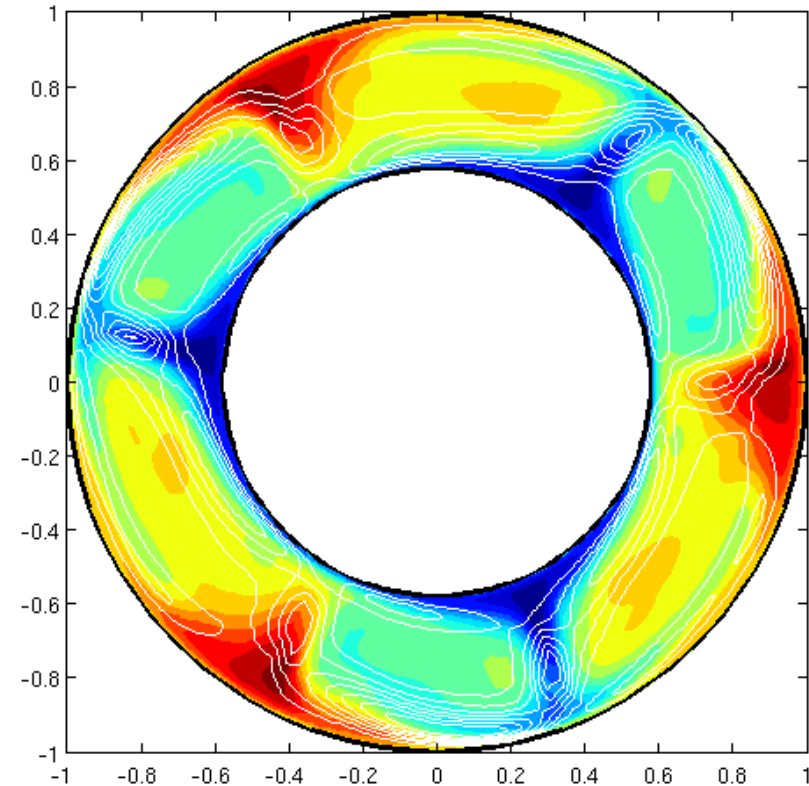
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Horizontal structure

- Azimuthal mode 3:

  three pairs of convection cells
  each with a hot and a cold jet

- Thin thermal boundary layers

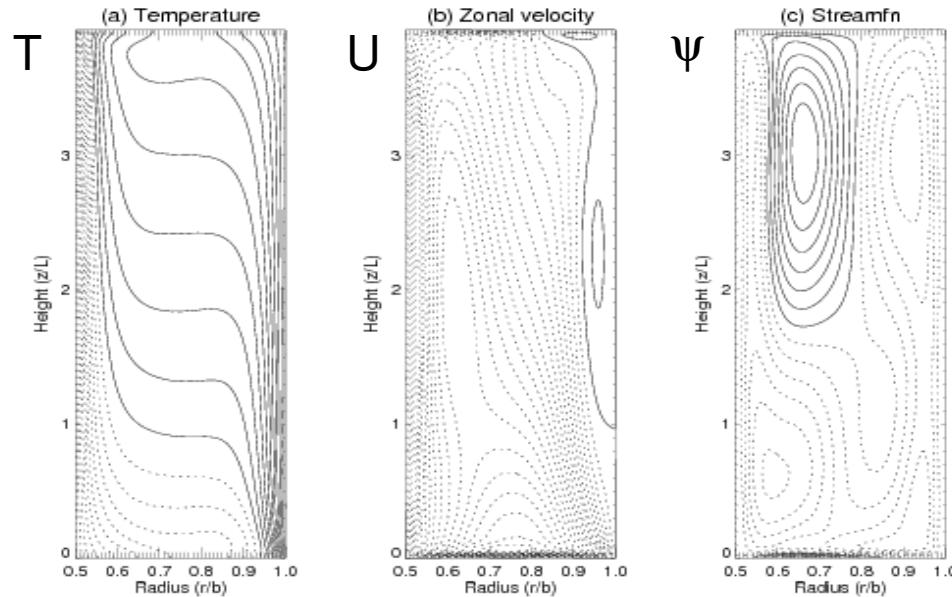- Wider velocity boundary layers

- Very narrow radial jets

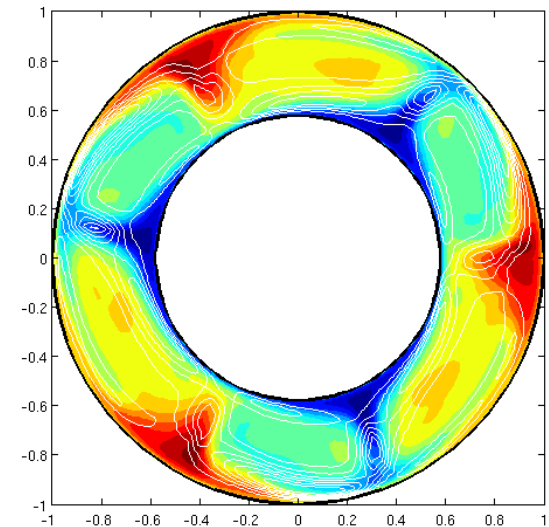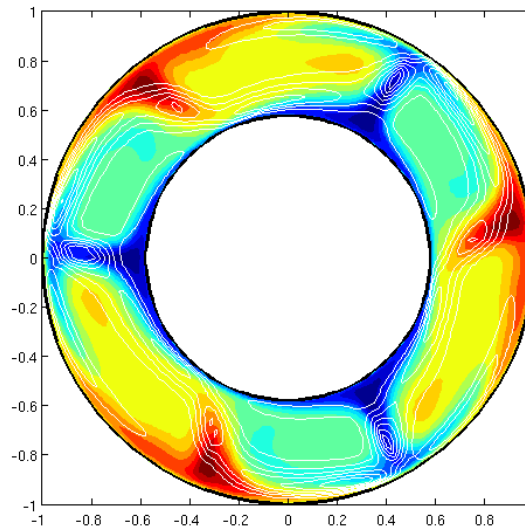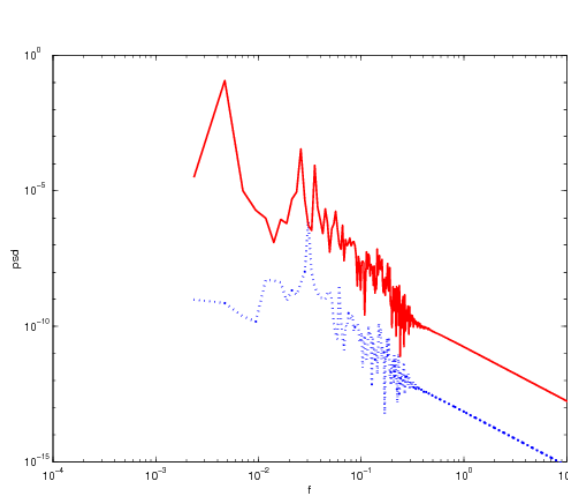*Colour: heat*

Contours: kinetic energy

Radial cross-sections

- Largely barotropic

  with flat isotherms in interior and horizontal shear
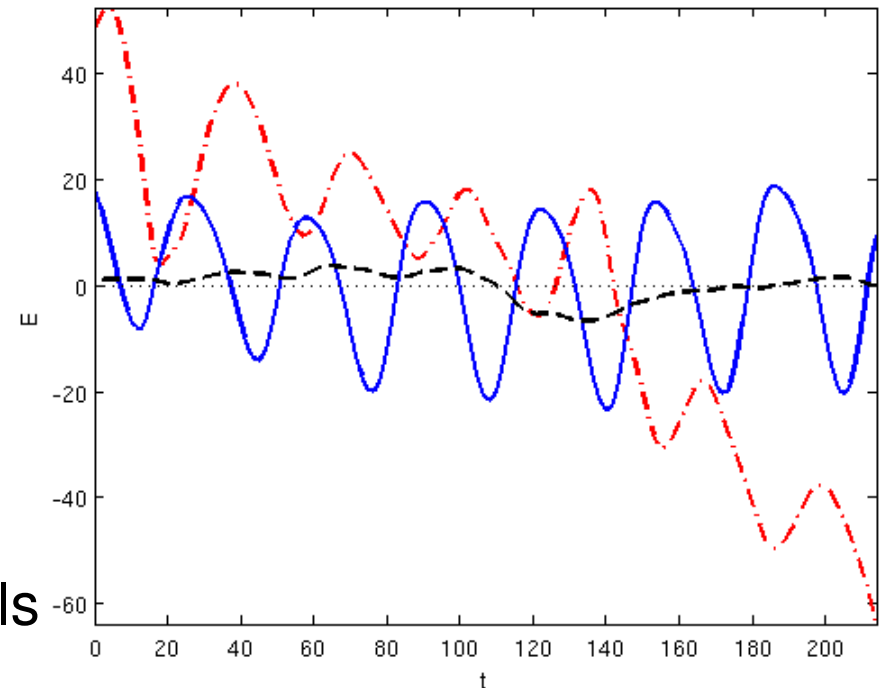
Onset of temporal variation

- Drifting large-scale wave appears 'almost steady'

- Spectra of show broad spectral peak beside pattern drift

- Shape of jets fluctuating
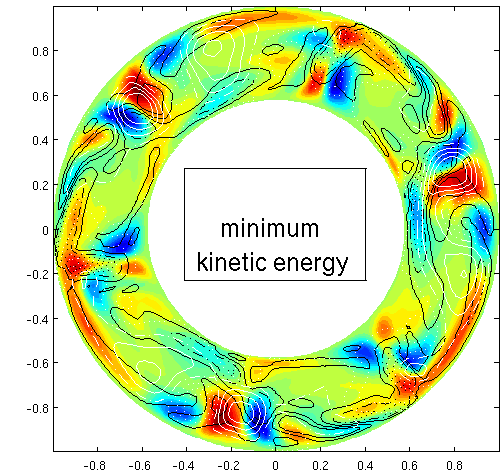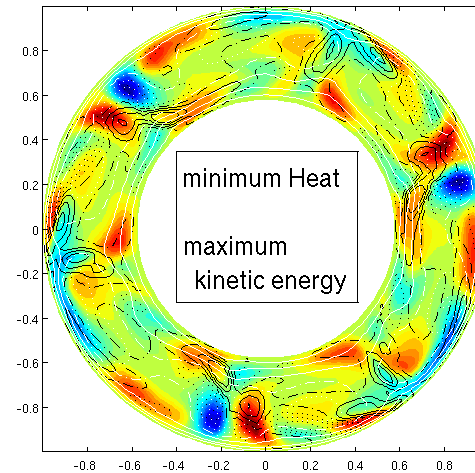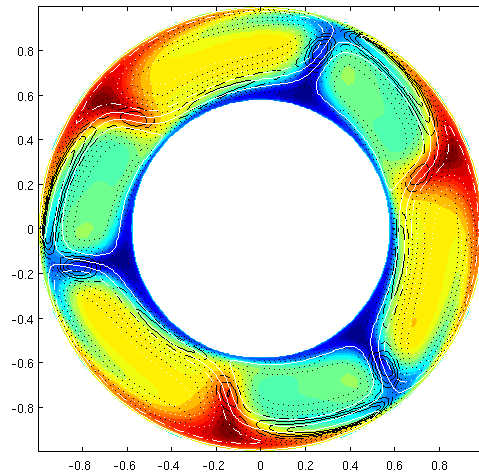
Evolution of volume-integrated energy

- Heat,

- Kinetic energy and

- Pressure energy

show

- slow modulation and

- modulated oscillation with a
  clear period

Total energy budget is balanced by
fluctuating heat flux through side walls

Time mean and fluctuation fields

- Move into frame co-rotating with dominant wave
  - Calculate time mean field
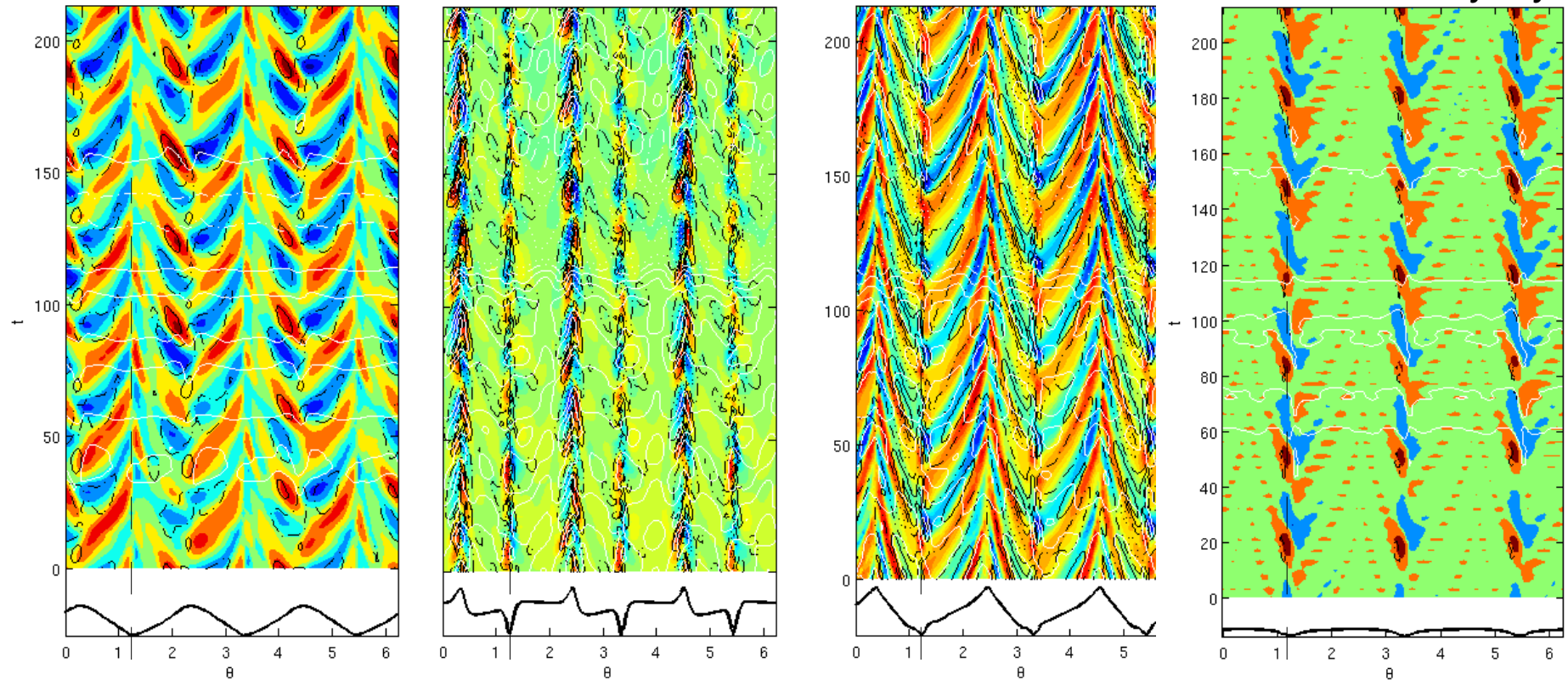 - and fluctuating fields

minimum Heat

maximum
kinetic energy

minimum
kinetic energy

Time evolution at four radial positions
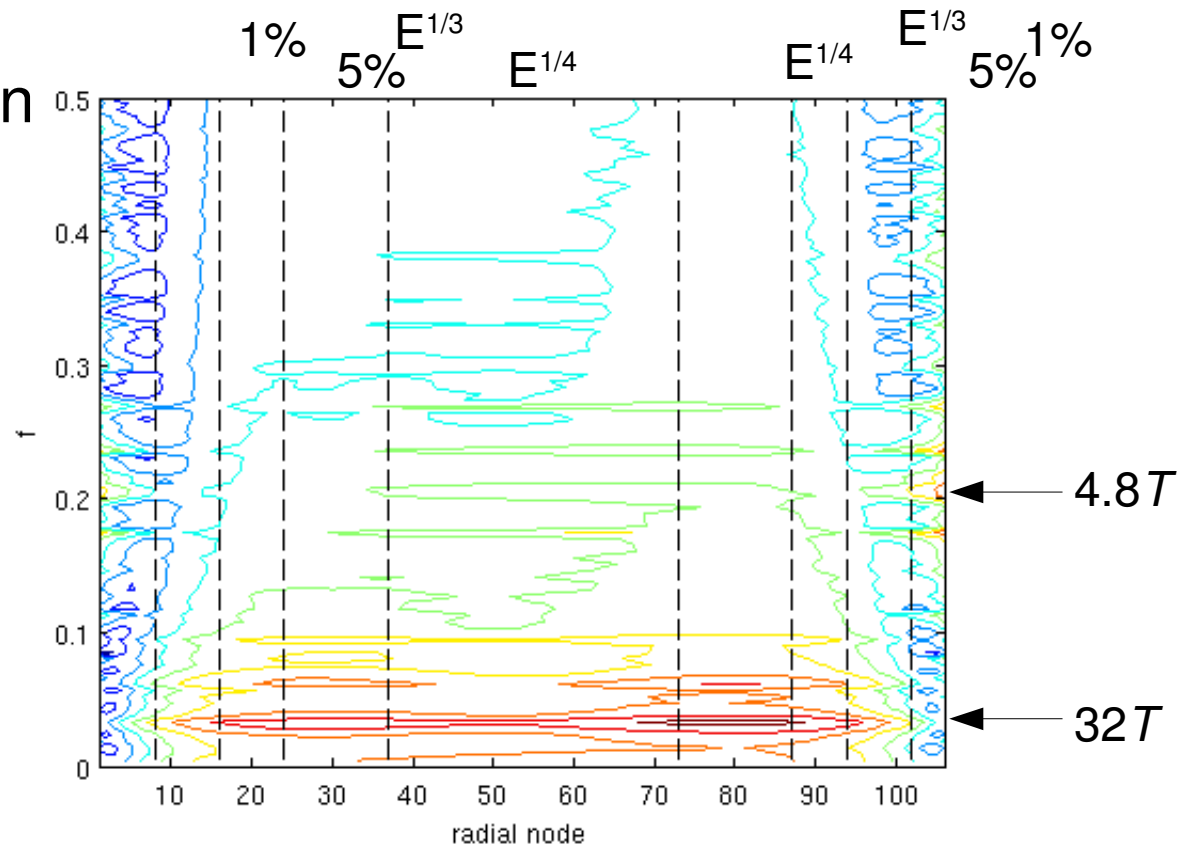

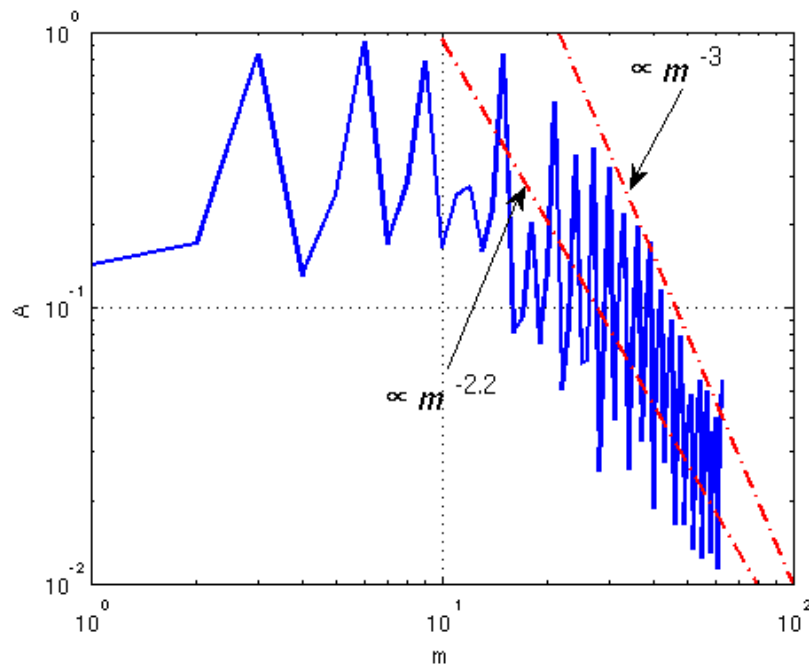Cold inner side | centre | Warm side | Warm boundary layer

Spectra against radius; azimuthally averaged

- Strongest activity within quasi-geostrophic $E^{1/4}$ Stewartson layer at lower frequency

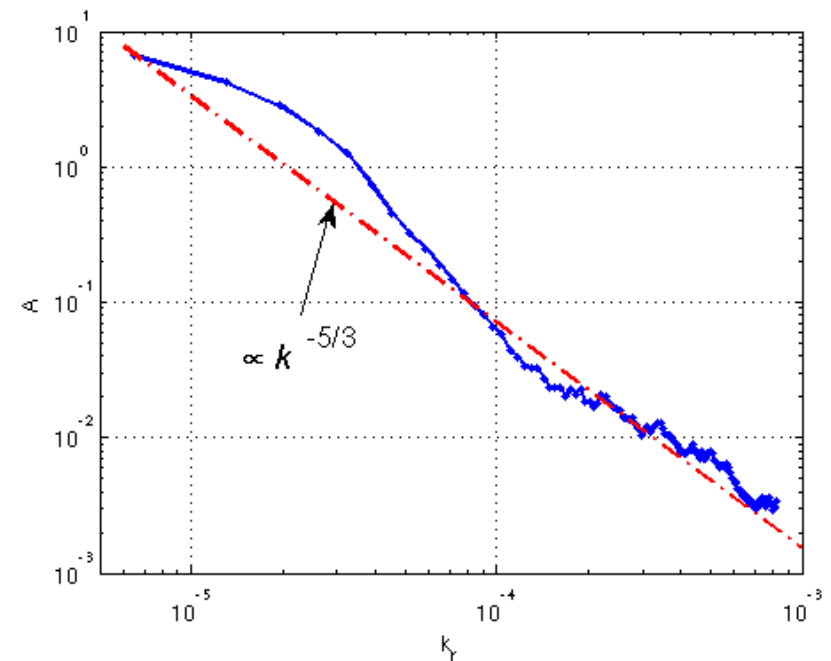- High-frequency variability strongest within 1% of width

Spatial spectra

- Azimuthal spectra,
  averaged over time & radius

- Radial spectra,
  averaged overtime & azimuth

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Summary

- Large scale flow structure of a slowly drifting (214T) almost steady mode 3 of three pairs of radial jets

- Jets fluctuate at time scale of ~31T, esp. on arrival at other side

- In boundary layer, fast fluctuations (~ 4.8T) are observable, especially between jets

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- Azimuthal structure dominated by large-scale structure

- Radial structure less obvious, has smooth spectrum $\propto k^{-5/3}$

Conclusions (?)

- Onset of fluctuations immediately aperiodic (weakly turbulent?)

- From single period to three periodic components

- Jets fluctuating, esp. pulsation at root and spreading at tip
  in outer Stewartson layers

- Fast fluctuations generated at boundaries move into interior
  (cf Plenary by G van Heijst on Tuesday and sessions 1.4,1.6)

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- Boundary layers are crucial part