

# Elastic turbulence in 2D viscoelastic flows

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# Outline

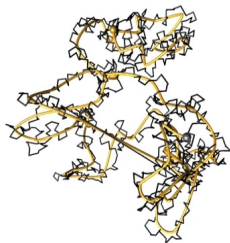
- 1 Elastic turbulence
- 2 Hydrodynamic model
- 3 DNS of elastic turbulence
- 4 Conclusions

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# Viscoelastic fluids

## Solutions of flexible long-chain polymers



$C \sim (1 \div 10) ppm$  in weight (dilute solutions)

$\tau \sim (1 \div 10)s$  (slowest) relaxation time

## Striking effects on flowing fluids

$$Wi \equiv \frac{U\tau}{L}$$

Weissenberg number

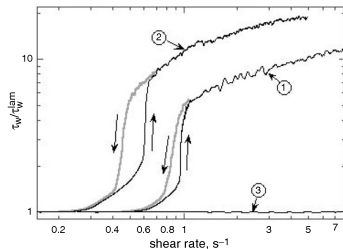
$$Re \equiv \frac{UL}{\nu}$$

Reynolds number

$Re \simeq 0; Wi \gg 1 \Rightarrow$  Elastic turbulence

# Some experimental results

## Flow resistance

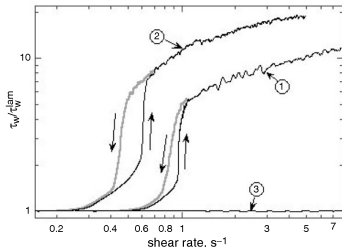


- swirling flow between two parallel disks
- high m.w. polyacrylamide
- dilute solution in a viscous sugar syrup

A. Groisman, V. Steinberg, Nature **405**, 53 (2000)

# Some experimental results

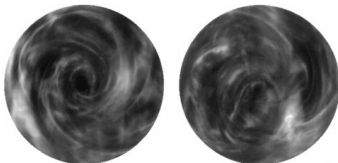
## Flow resistance



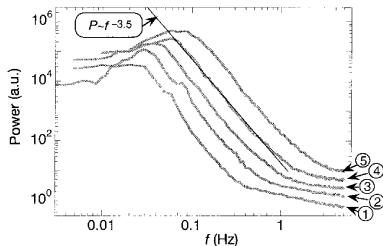
- swirling flow between two parallel disks
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## Snapshots of the flow

$Wi = 13$   
 $Re = 0.7$



## Spectra of velocity fluctuations



A. Groisman, V. Steinberg, Nature **405**, 53 (2000)

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# Oldroyd-B model

$$\left\{ \begin{array}{l} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nu \Delta \mathbf{u} + \mathbf{f} + \underbrace{\frac{2\eta\nu}{\tau} \nabla \cdot \boldsymbol{\sigma}}_{\text{feedback}} \\ \partial_t \boldsymbol{\sigma} + (\mathbf{u} \cdot \nabla) \boldsymbol{\sigma} = \underbrace{(\nabla \mathbf{u})^T \cdot \boldsymbol{\sigma} + \boldsymbol{\sigma} \cdot (\nabla \mathbf{u})}_{\text{stretching}} - \underbrace{\frac{2}{\tau}(\boldsymbol{\sigma} - \mathbf{1})}_{\text{relaxation}} \end{array} \right.$$

where...

$$\sigma_{ij} \equiv \frac{\langle R_i R_j \rangle}{R_0^2}$$

$$\mathbf{R} = (R_1, \dots, R_d)$$

$$R_0$$

polymer conformation tensor

polymer end-to-end separation

radius of gyration



# Viscoelastic Kolmogorov flow (1)

Let us consider a 2D **parallel flow**

$$\mathbf{f} = (F \cos(y/L), 0) \quad \text{Kolmogorov forcing}$$

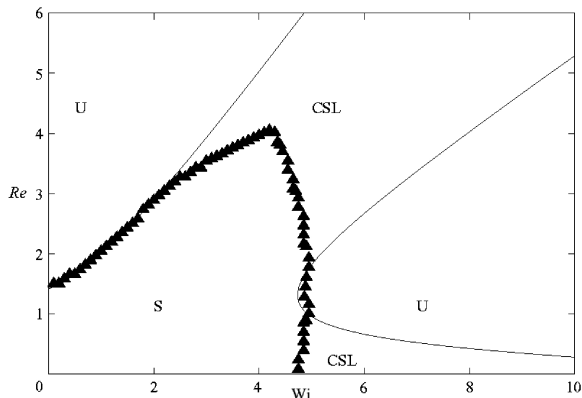
Laminar fixed point:

$$\mathbf{u} = (U_0 \cos(y/L), 0)$$

$$\boldsymbol{\sigma} = \begin{pmatrix} 1 + \tau^2 \frac{U_0^2}{2L^2} \sin^2(y/L) & -\tau \frac{U_0}{2L} \sin(y/L) \\ -\tau \frac{U_0}{2L} \sin(y/L) & 1 \end{pmatrix}$$

$$F = [\nu U_0(1 + \eta)]/L^2; \quad Wi \equiv \frac{U_0 \tau}{L}; \quad Re \equiv \frac{U_0 L}{\nu(1 + \eta)}$$

# Viscoelastic Kolmogorov flow (2)



G. Boffetta, A. Celani, A. Mazzino, A. Puliafito, M. Vergassola, J. Fluid Mech. **523**, 161 (2005)

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# Cholesky decomposition

## Numerical integration:

2D Oldroyd-B  $\Rightarrow \omega, \sigma_{11}, \sigma_{22}, \sigma_{21} = \sigma_{12}$

$\sigma$  is **positive definite**  $\Rightarrow \sigma = \mathbf{L}\mathbf{L}^T$

$\mathbf{L}$  is a lower triangular matrix:

$$\ell_{11} = \sqrt{\sigma_{11}}$$

$$\ell_{21} = \frac{\sigma_{12}}{\sqrt{\sigma_{11}}} = \frac{\sigma_{12}}{\ell_{11}}$$

$$\ell_{22} = \sqrt{\sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}}} = \frac{\det \sigma}{\ell_{11}}$$

T. Vaithianathan, L. R. Collins, J. Comp. Physics **187**, 1 (2003)

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$$\boxed{\tilde{\ell}_{ij} \rightarrow \ell_{ij} = e^{\tilde{\ell}_{ij}}; \ell_{ij} = \tilde{\ell}_{ij} \rightarrow \sigma = \mathbf{L}\mathbf{L}^T}$$

T. Vaithianathan, L. R. Collins, J. Comp. Physics **187**, 1 (2003)

# Momentum budget

$$\partial_y \langle u_x u_y \rangle = \nu \partial_y^2 \langle u_x \rangle + \frac{2\nu\eta}{\tau} \langle \sigma_{xy} \rangle + f_x$$

$$\Pi_R \equiv \langle u_x u_y \rangle; \Pi_p \equiv \frac{2\nu\eta}{\tau} \langle \sigma_{xy} \rangle; \Pi_\nu \equiv \nu \partial_y \langle u_x \rangle$$

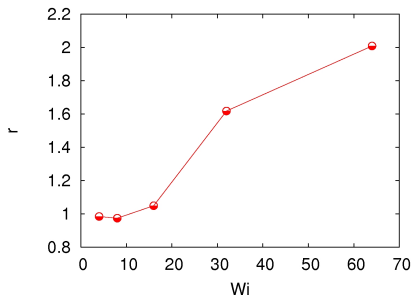
$$r \equiv \frac{\Pi}{\Pi_{lam}} = \frac{\Pi_R + \Pi_p}{\Pi_{lam}} = \frac{1+\eta}{FL\eta} U_2 + \frac{2\nu(1+\eta)}{FL\tau} \Sigma$$

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$$El \equiv \frac{Wi}{Re} = 64; Wi = F$$

$$\tau = 4; L = 1/4; \nu(1+\eta) = 1$$

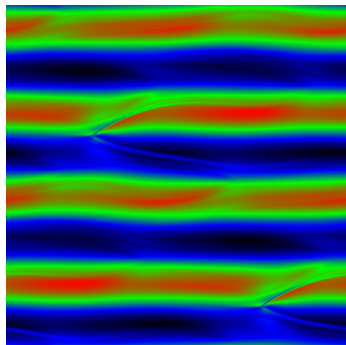
$\frac{\Pi_R}{\Pi_{lam}} < 10^{-2} \implies$  the "turbulent" stress is due to **elasticity**



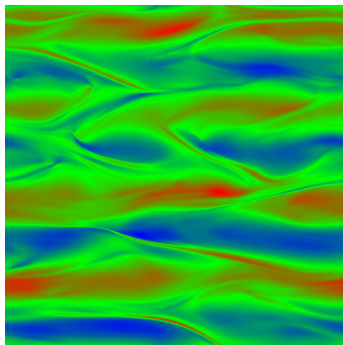
# Chaotic flow (1)

## Snapshots of the vorticity field

$Re = 1; Wi = 16$



$Re = 1; Wi = 64$

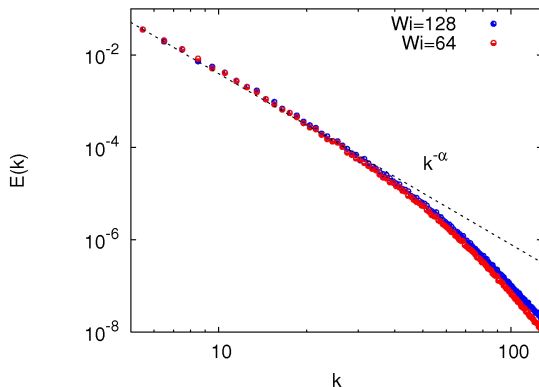


- $\omega_{max} = U/L$
- $\omega = 0$
- $\omega_{min} = -U/L$

The flow develops active modes at all the scales

## Chaotic flow (2)

### Spectra of velocity fluctuations in the turbulent-like states

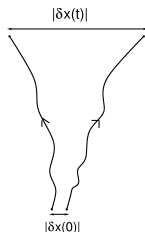


$$E(k) \sim k^{-\alpha}$$

$$3 < \alpha < 4$$

Elastic turbulence corresponds to a **smooth** chaotic flow

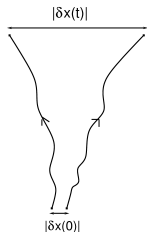
# Mixing: Lagrangian Lyapunov exponent



$$\lambda_L \equiv \lim_{t \rightarrow \infty} \lim_{|\delta \mathbf{x}(0)| \rightarrow 0} \frac{1}{t} \ln \frac{|\delta \mathbf{x}(t)|}{|\delta \mathbf{x}(0)|}$$

$$G(\gamma) : P_t(\gamma) \sim e^{-t G(\gamma)}$$

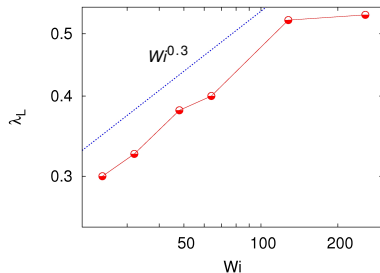
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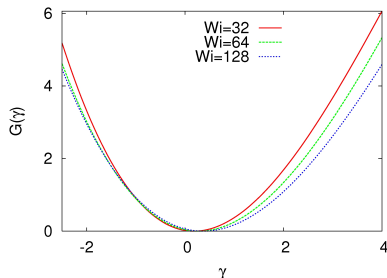
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Lyapunov exponent



Cramer function



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## Conclusions

The basic phenomenology of elastic turbulence is reproduced in  $2D$ .  
For  $Wi > W_c \sim 10$ :

- 1 the flow resistance grows with respect to the laminar case;
- 2 the flow displays features of a strongly non-linear state;
- 3 the Lagrangian Lyapunov exponent is positive and grows with  $Wi$ ; the distribution of its fluctuations becomes asymmetric.