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Kinetic Energy Repartition in MHD Turbulence

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Quasi-Static MHD Turbulence

MHD Equations:

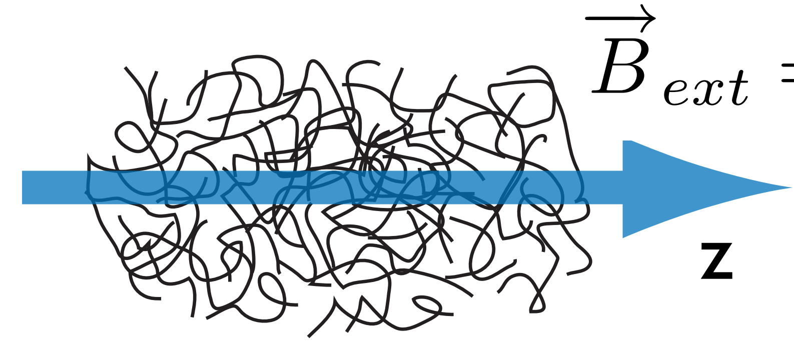
$$\partial_t B_i + u_j \partial_j B_i = B_j \partial_j u_i + \eta \Delta B_i$$

$$\partial_t u_i + u_j \partial_j u_i = -\partial_i p + \nu \Delta u_i + \frac{1}{\rho}(j \times B)_i + F_i^e$$

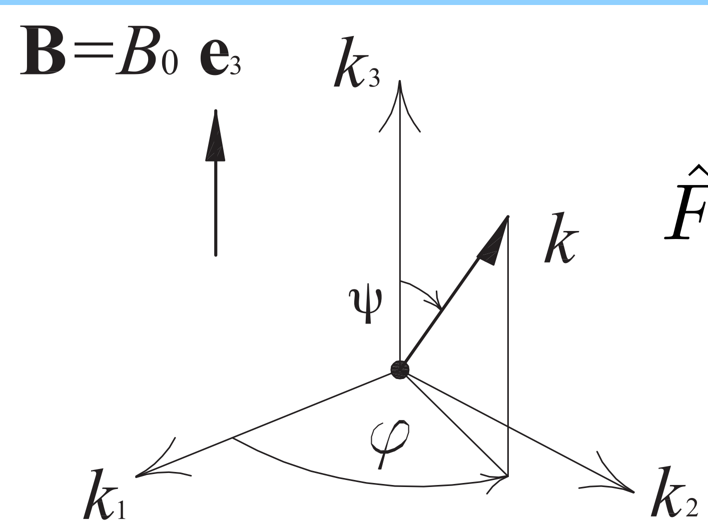
$\boxed{\text{If } R_\mathrm{m} \ll 1:}$

$$\cancel{\partial_t B_i + u_j \partial_j B_i = B_j \partial_j u_i + \eta \Delta B_i}$$

$$\vec{B}_{ext} = B_0 \vec{1}_z$$



$$\partial_t u_i + u_j \partial_j u_i = -\partial_i p + \nu \Delta u_i \underbrace{- \frac{\sigma B_0^2}{\rho} \Delta^{-1} \partial_z \partial_z u_i}_{\text{Lorentz force}} + F_i^e$$



Lorentz force

$$\hat{F}_i(\mathbf{k}) = -\frac{\sigma B_0^2}{\rho} \cos^2(\psi) \hat{u}_i(\mathbf{k})$$

(Fourier space)

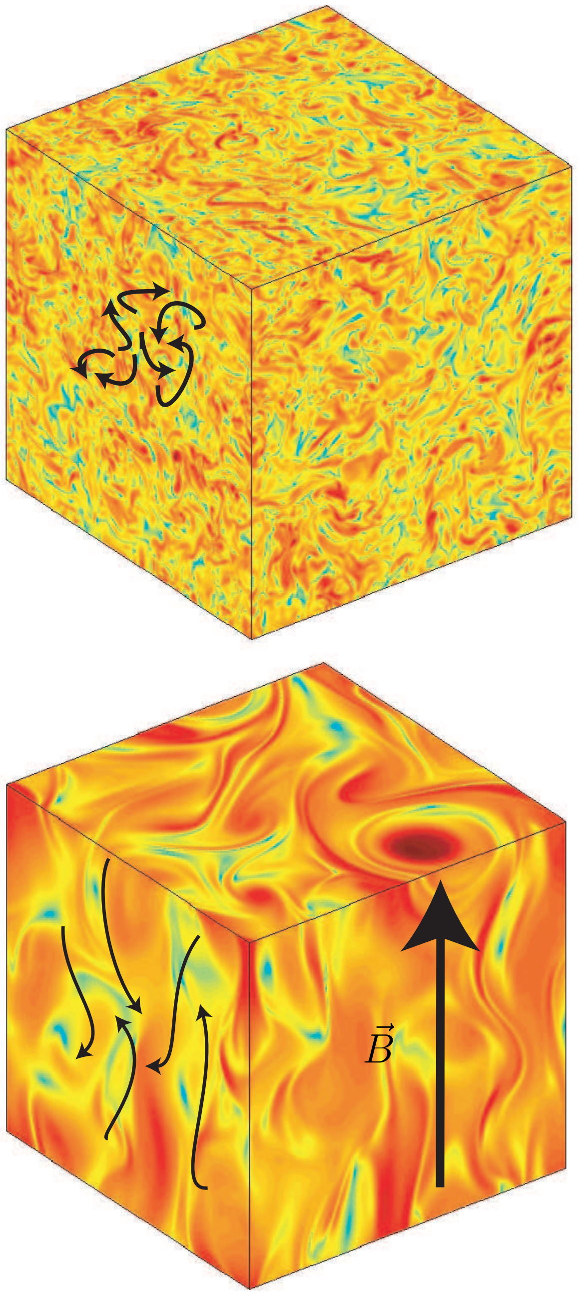
Interaction parameter: $N = \dfrac{\sigma B^2 u}{\rho L}$

Anisotropic Joule dissipation

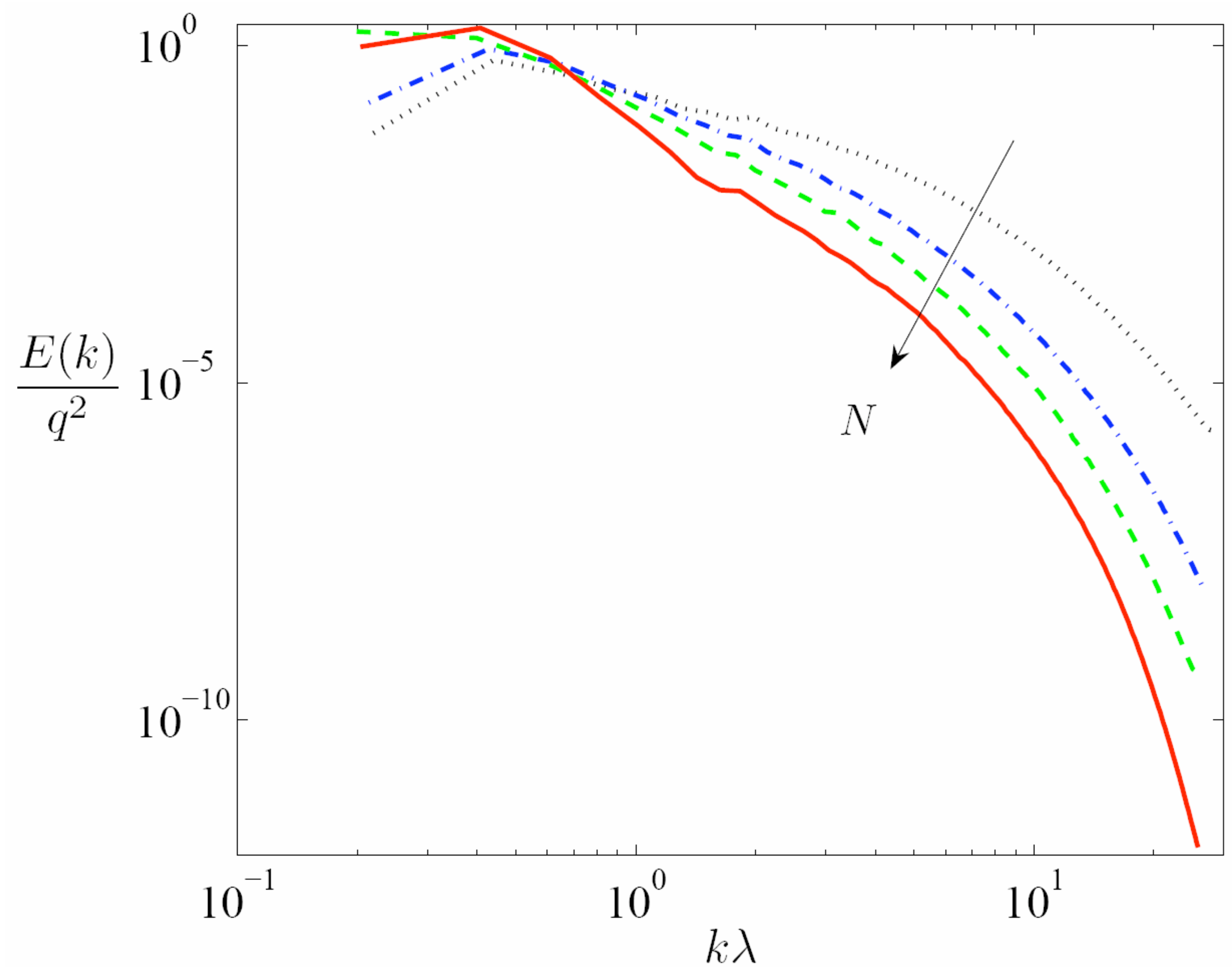
$$\hat{F}_i(\mathbf{k}) = -\frac{\sigma B_0^2}{\rho}\cos^2(\psi)\hat{u}_i(\mathbf{k})$$

Always dissipative

Anisotropic: maximum for $\psi = 0, \ (\mathbf{k} \| \mathbf{B})$

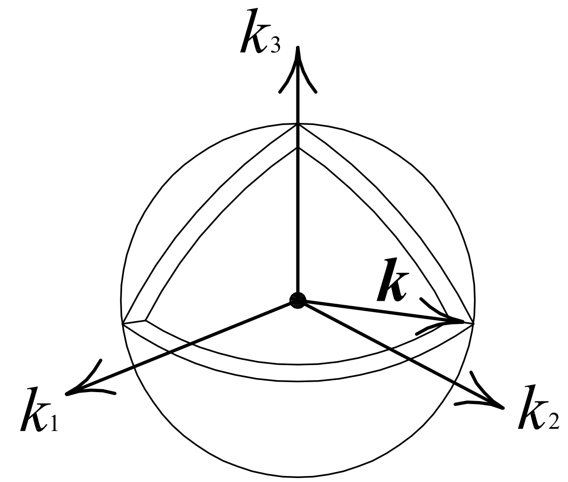


(contours of the vorticity in a periodic domain simulation, forced turbulence)



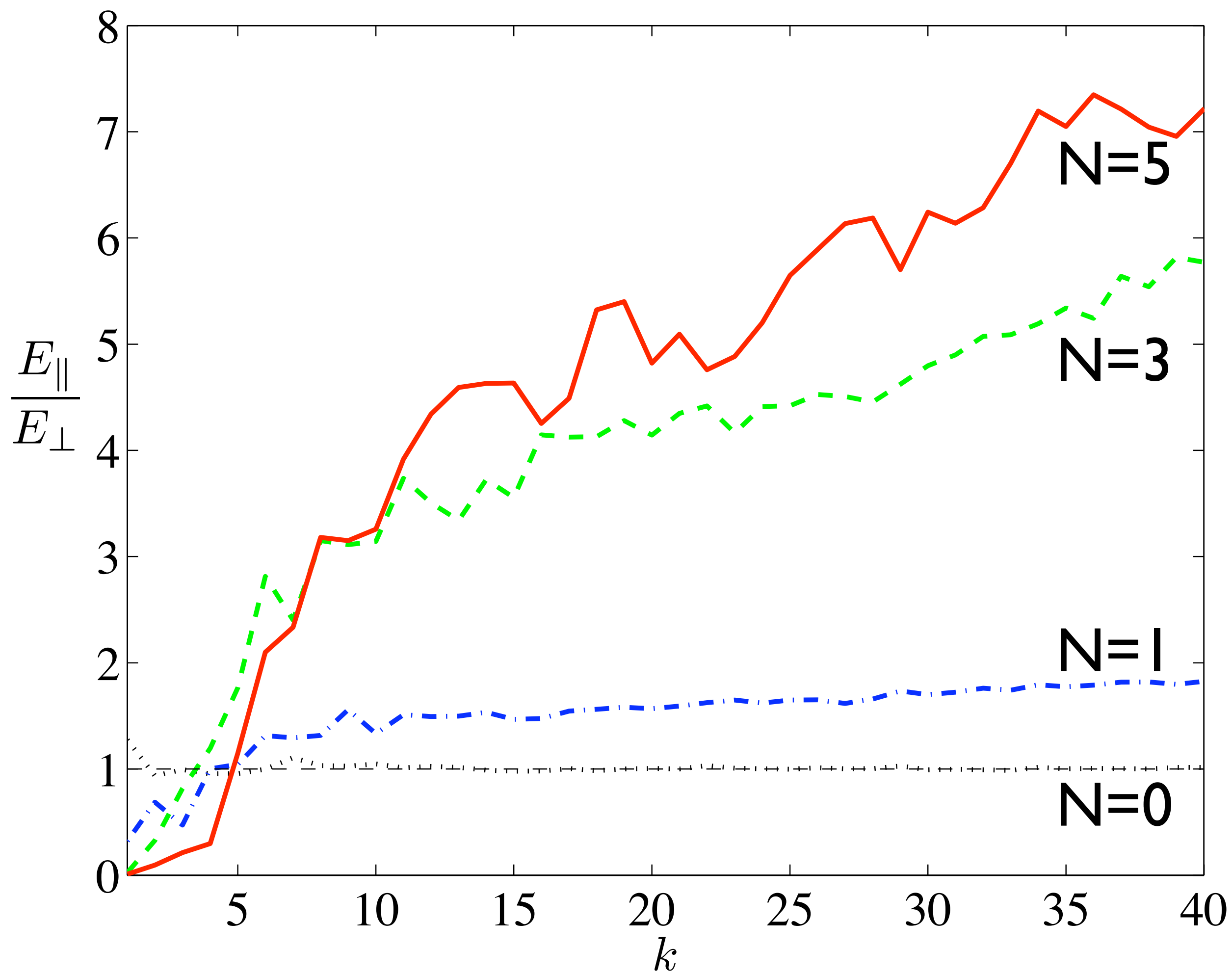
Scale-by-scale kinetic energy (I)

"Spherical averaging" $R_{ii}(\mathbf{r}) = \langle u_i(\mathbf{x}) u_i(\mathbf{x}+\mathbf{r}) \rangle$



$$E_z(k) = E_{\parallel} = \frac{1}{2}\int_0^{\pi}\int_0^{2\pi} d\phi d\psi k^2 \sin(\psi) \|\hat{u}_z(k,\phi,\psi)\|^2$$

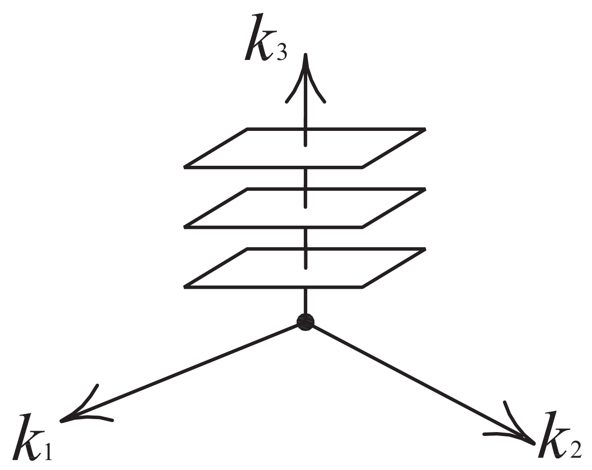
$$\left.\begin{array}{l} E_x(k) = \ldots \\ E_y(k) = \ldots \end{array}\right\} \quad E_{\perp}(k) = \frac{E_x(k) + E_y(k)}{2}$$



Same results as observed in Vorobev et al. Phys. Fluids 2005 (17) 125105

Scale-by-scale kinetic energy (II)

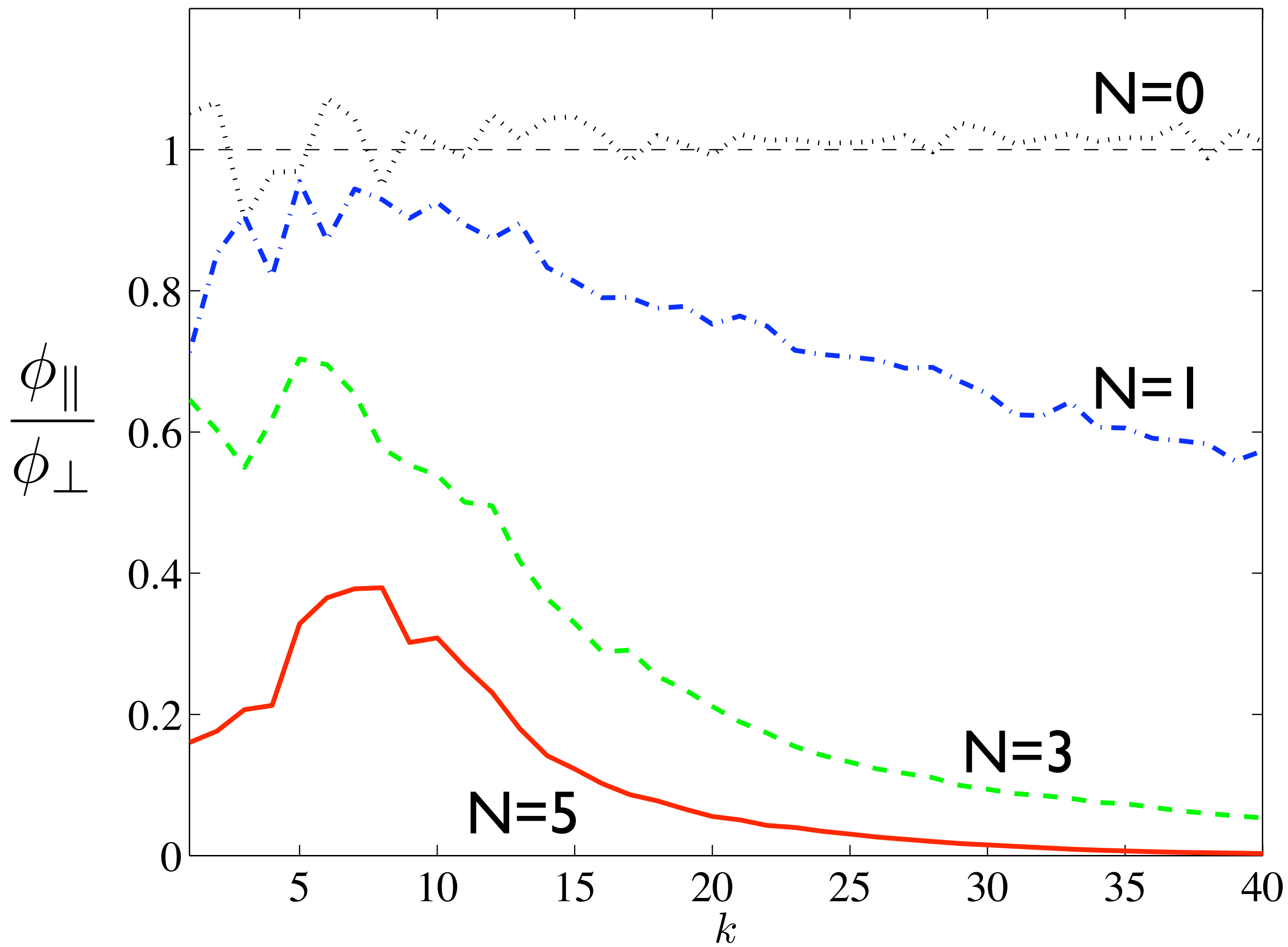
"Plane averaging"



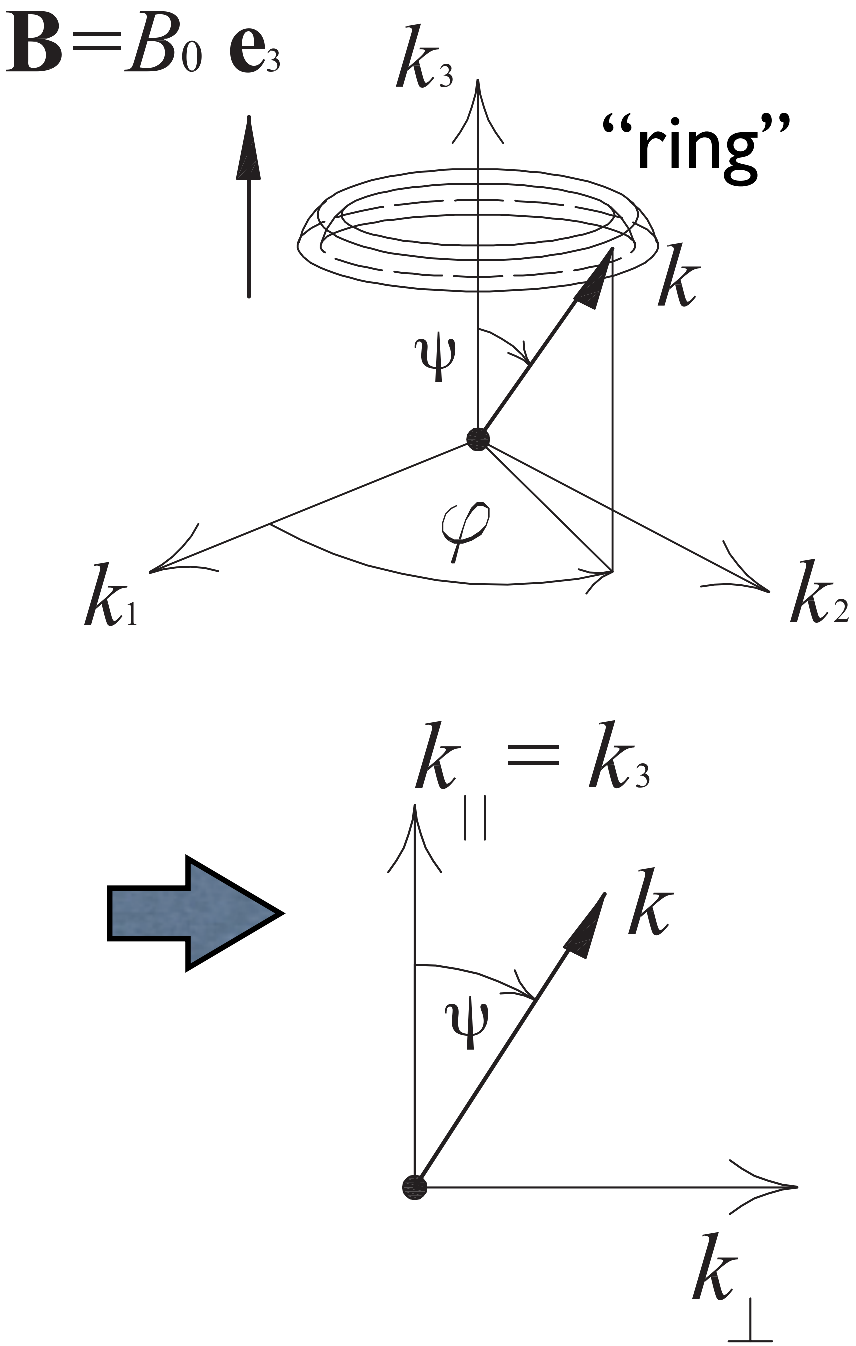
$$Q_{zz}(z) = \langle u_z(\mathbf{x}) u_z(\mathbf{x} + z\mathbf{e_z}) \rangle \quad \text{(experimental measure)}$$

$$\phi_{zz}(k_z) = \phi_{\|}(k_{\|}) = \int_0^{\infty}\int_0^{\infty} dk_x dk_y \|\hat{u}_z(k_x, k_y, k_z)\|^2$$

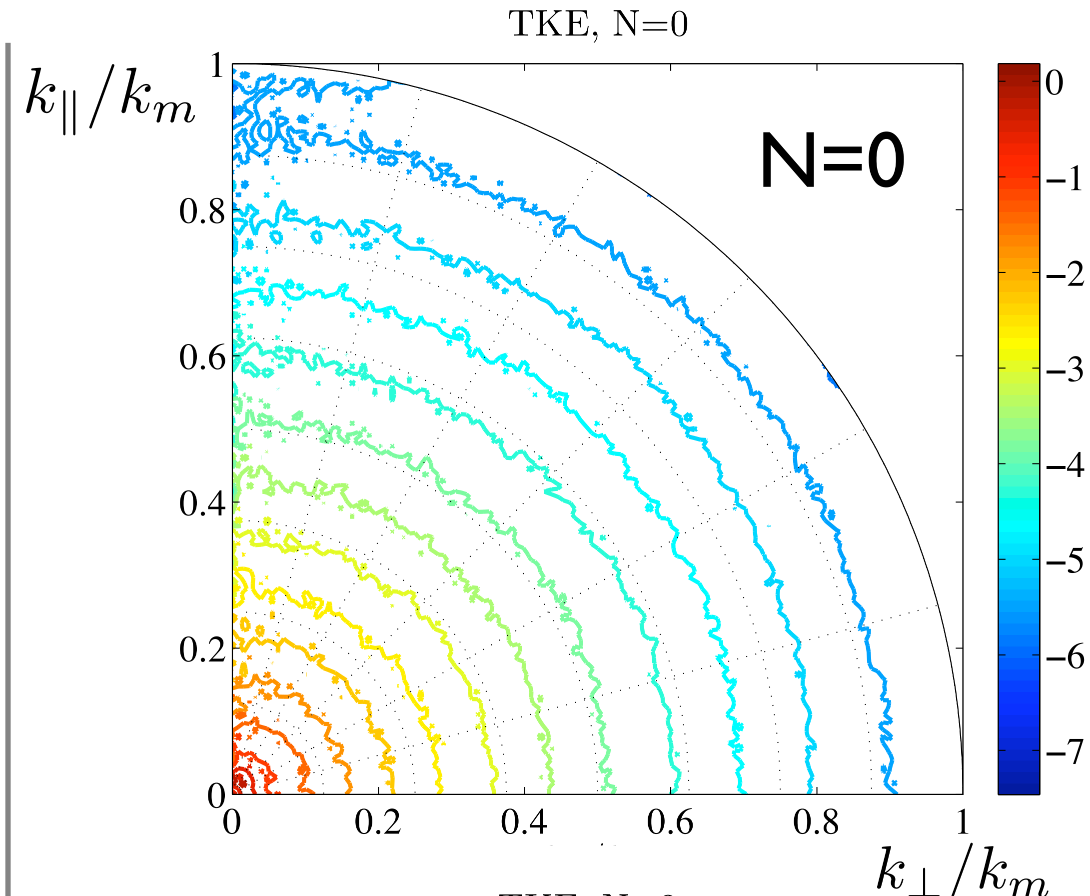
$$\left.\begin{array}{l} \phi_{yy}(k_y) = \ldots \\ \phi_{xx}(k_x) = \ldots \end{array}\right\} \quad \phi_{\perp}(k_{\perp}) = \frac{\phi_{yy}(k_{\perp}) + \phi_{xx}(k_{\perp})}{2}$$

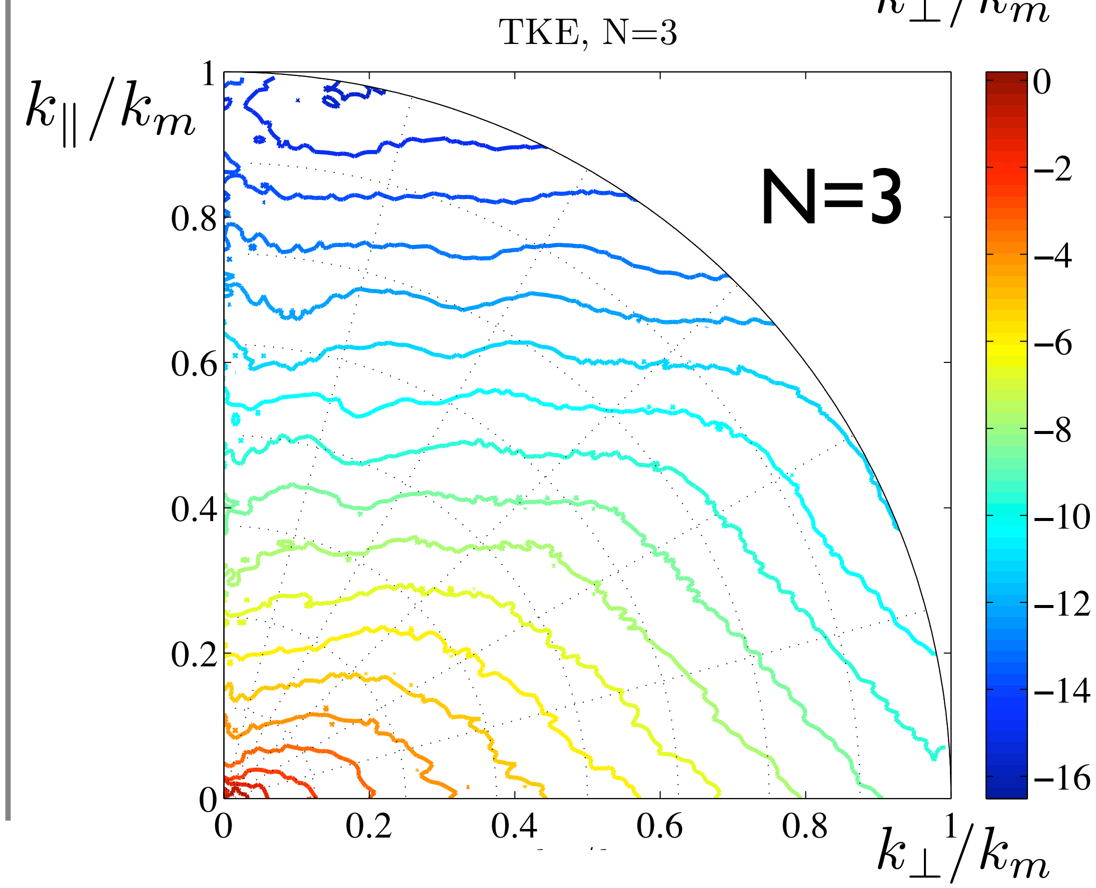


Kinetic energy distribution in spectral space
"Ring" averaging



$$\langle E \rangle_{\text{ring}} = \frac{1}{V_{\text{ring}}} \int_{\text{ring}} dV \, \|\hat{\mathbf{u}}(k_x, k_y, k_z)\|^2$$





Energy transfers

Momentum balance:

$$\partial_t u_i + u_j \partial_j u_i = -\partial_i p + \nu \Delta u_i - \frac{\sigma B_0^2}{\rho} \Delta^{-1} \partial_z \partial_z u_i + F_i^e$$

Energy balance (statistical equilibrium):

$$F^e(\mathbf{k}) = T(\mathbf{k}) - 2\nu k^2 E(\mathbf{k}) - \frac{\sigma B_0^2}{\rho} \cos^2(\psi) E(\mathbf{k})$$

Energy injection (forcing)

Non-linear transfer

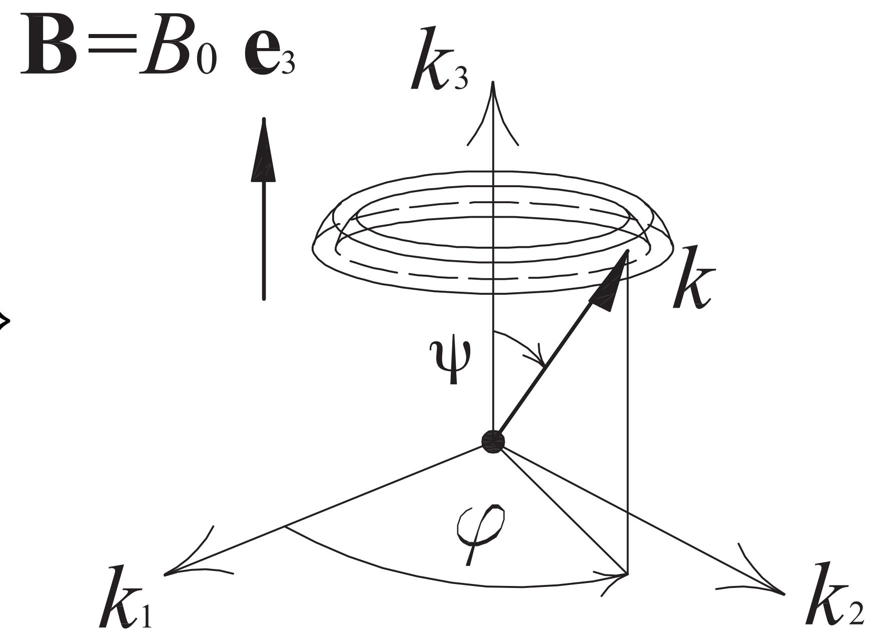
Molecular dissipation

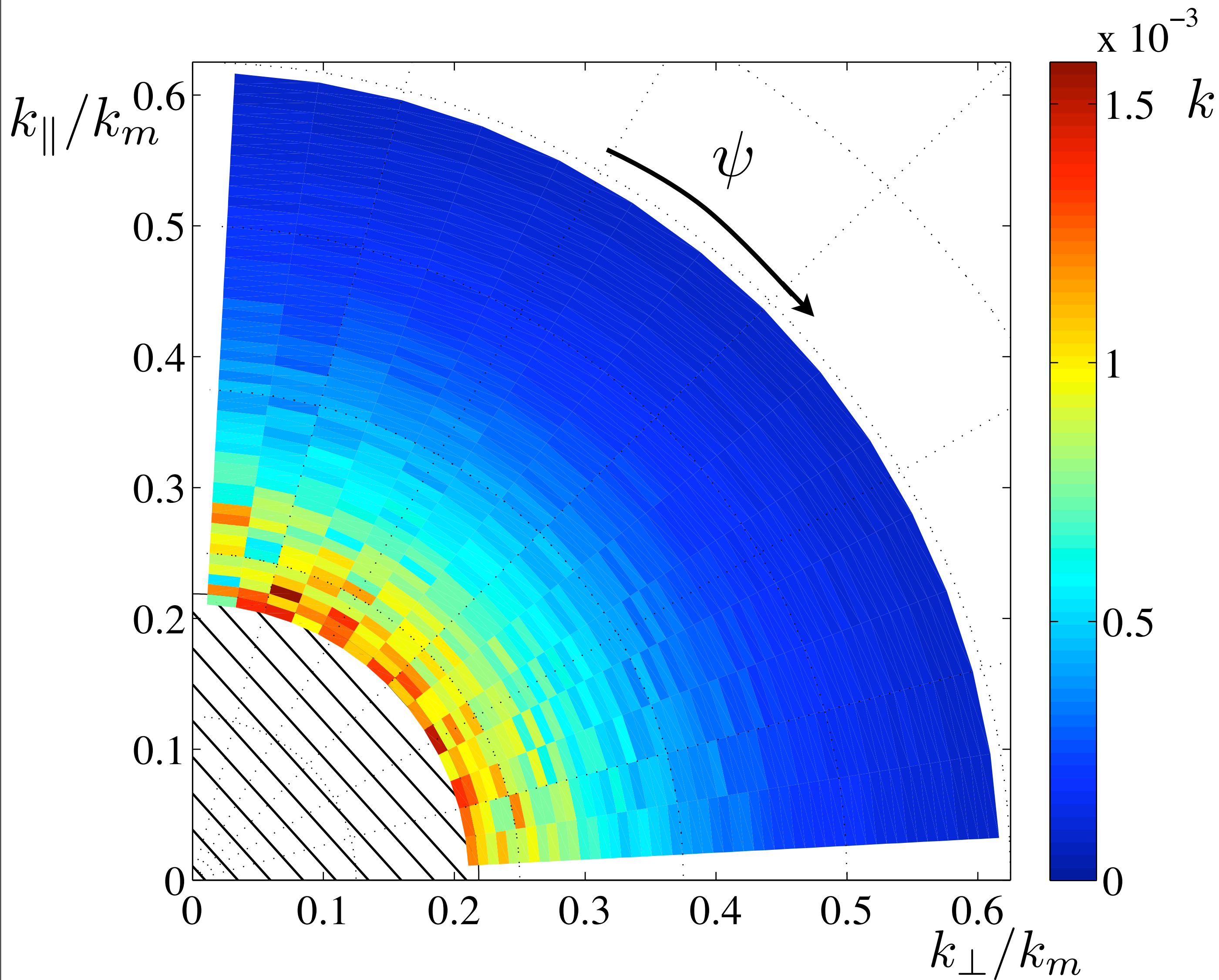
Joule dissipation

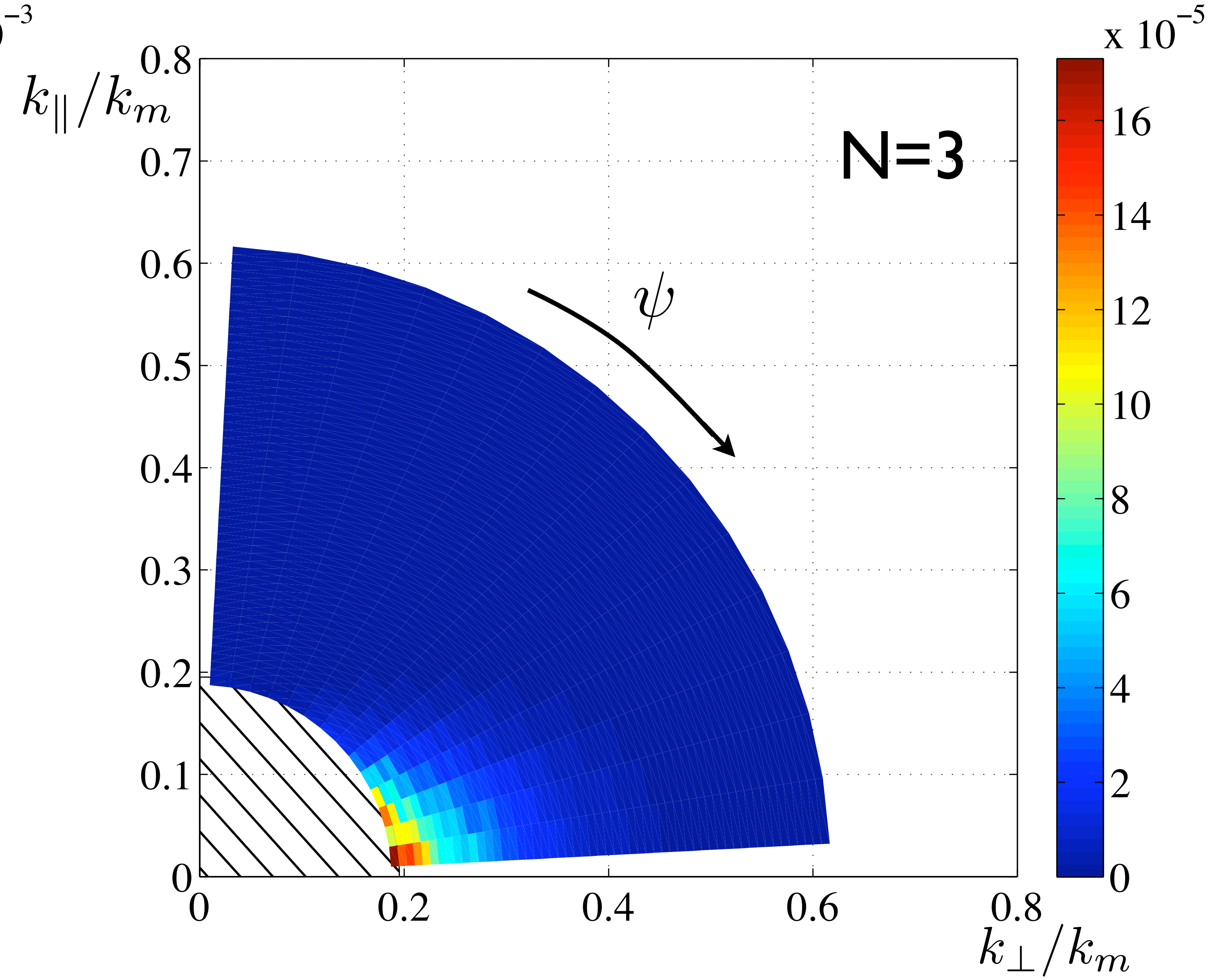
$T(\mathbf{k})$ represents the energy received by mode $\hat{\mathbf{u}}(\mathbf{k})$ from all the other modes.

Energy transfers
"Ring" transfers

$$T(\mathbf{k}) = k_l P_{jm}(\mathbf{k}) \mathrm{Re}\left\{ i\hat{u}_j(\mathbf{k}) \int \hat{u}_m^*(\mathbf{p}) \hat{u}_l^*(\mathbf{k}-\mathbf{p}) d\mathbf{p} \right\}$$

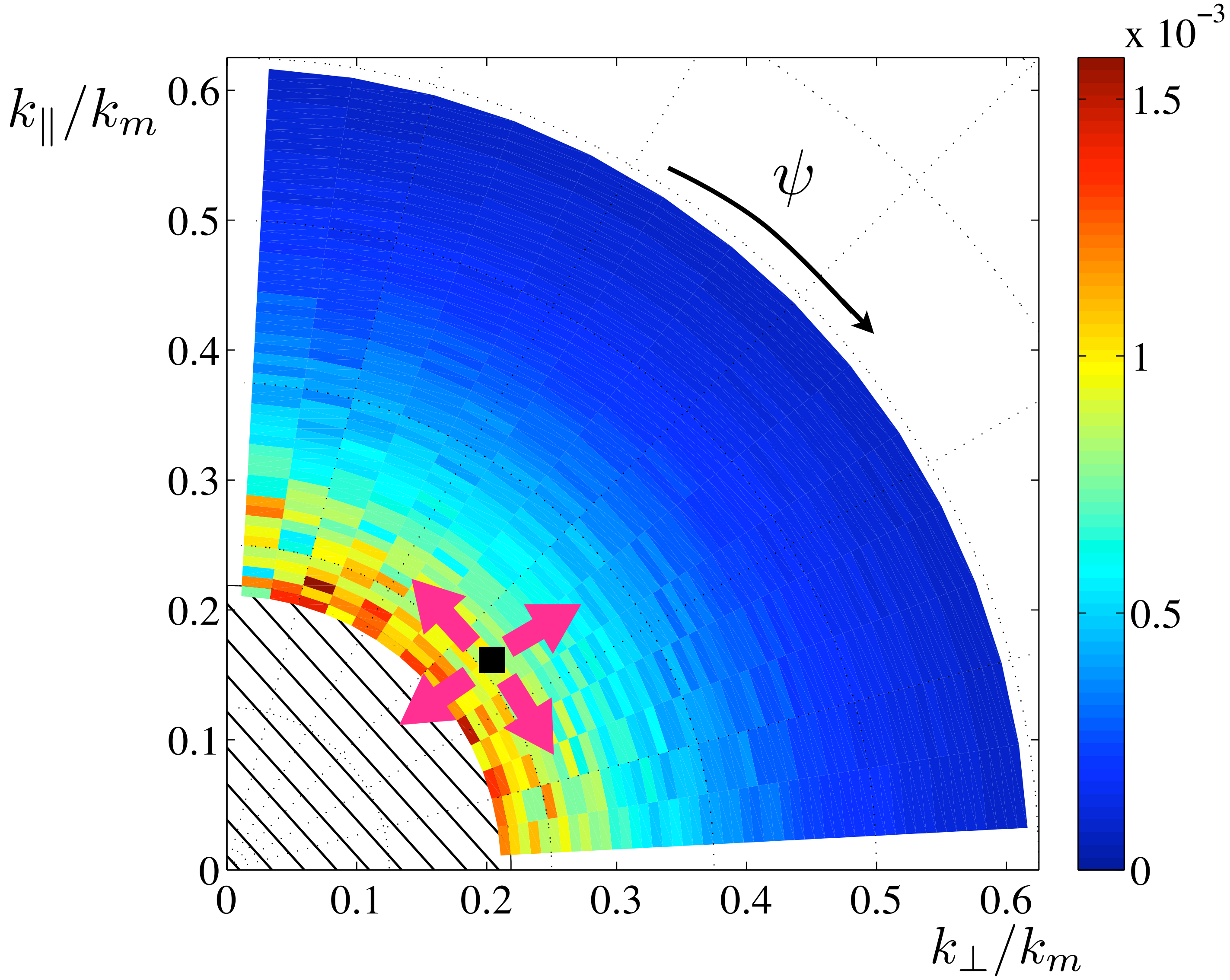


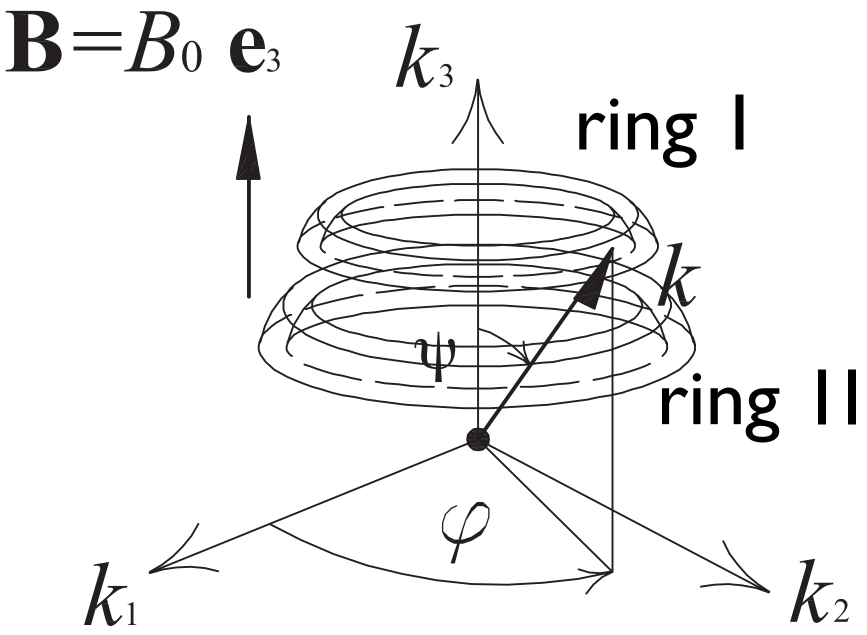




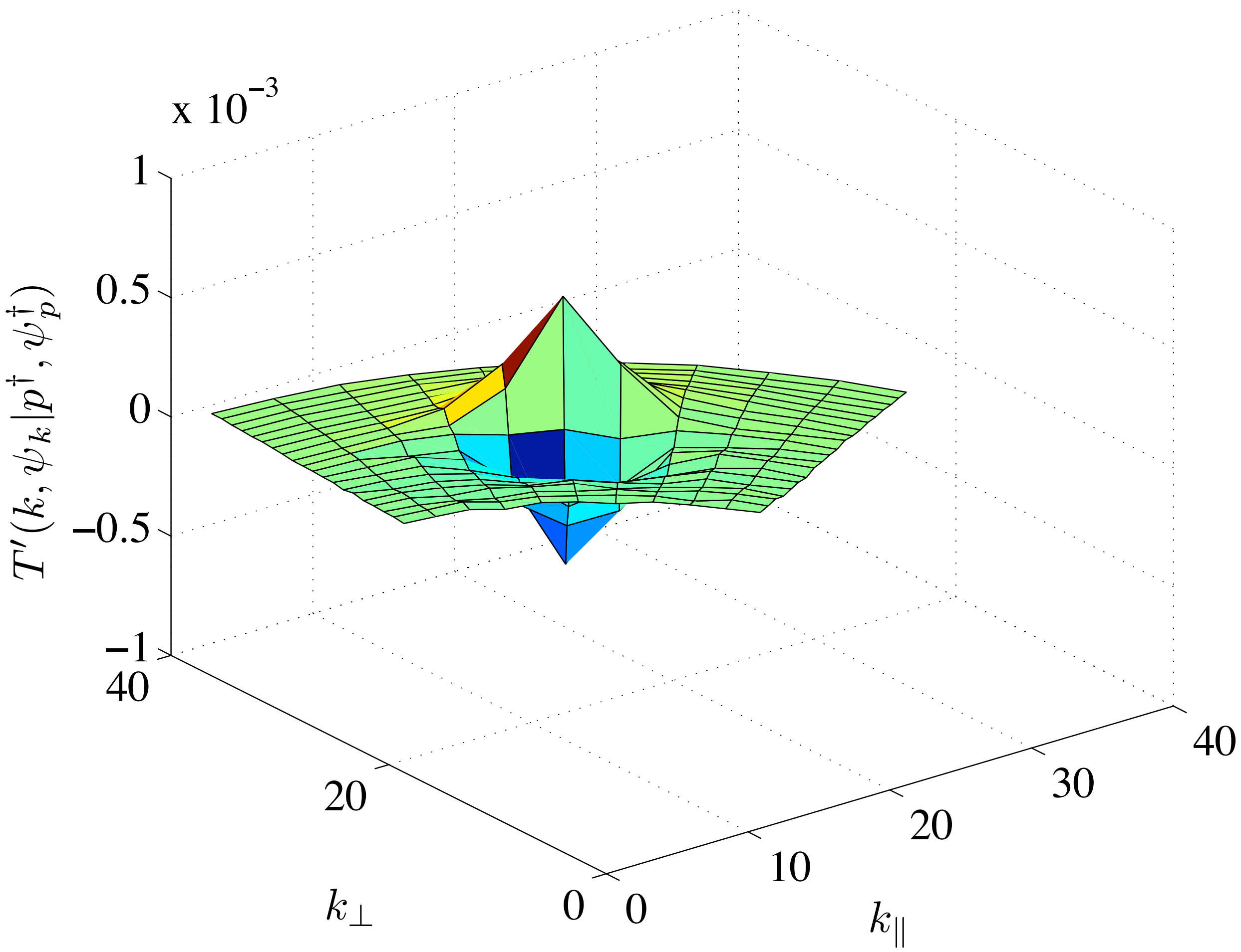
Ring to ring transfers (I)

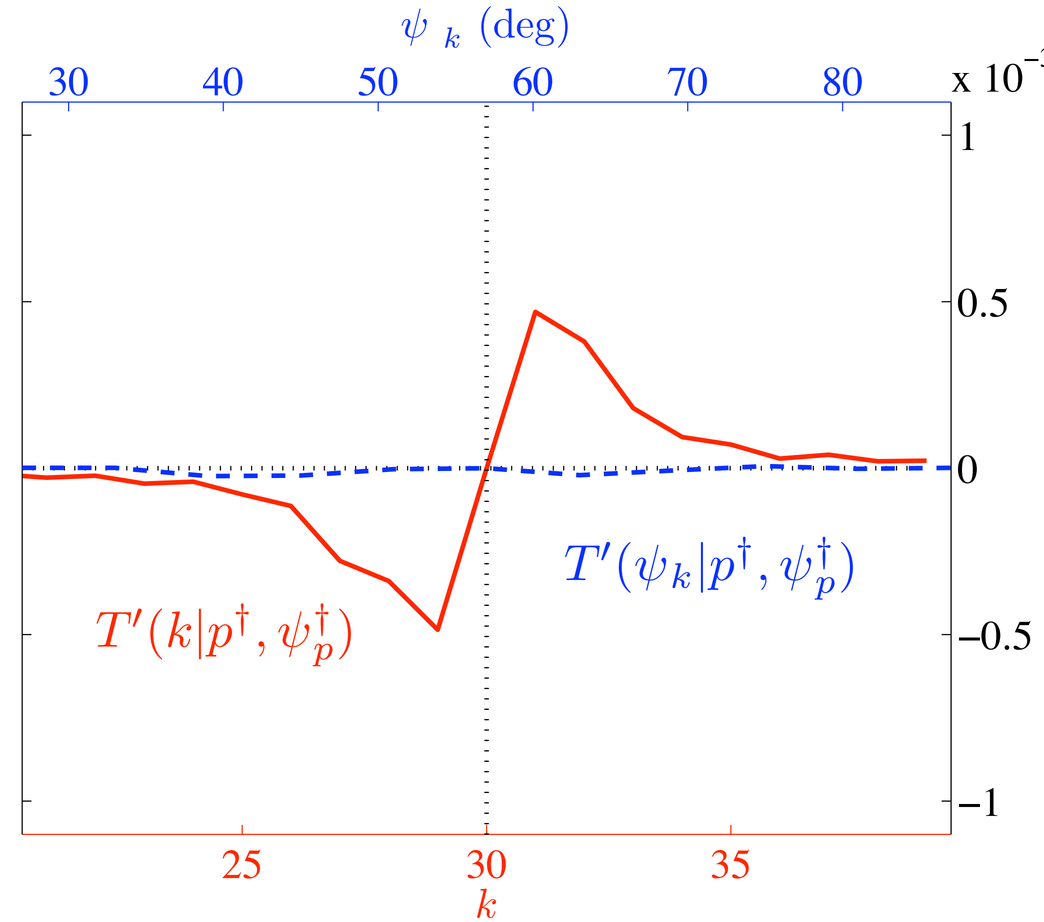
$$T(\mathbf{k}) = k_l P_{jm}(\mathbf{k}) \mathrm{Re} \left\{ i\hat{u}_j(\mathbf{k}) \int \hat{u}_m^*(\mathbf{p}) \hat{u}_l^*(\mathbf{k}-\mathbf{p}) d\mathbf{p} \right\}$$



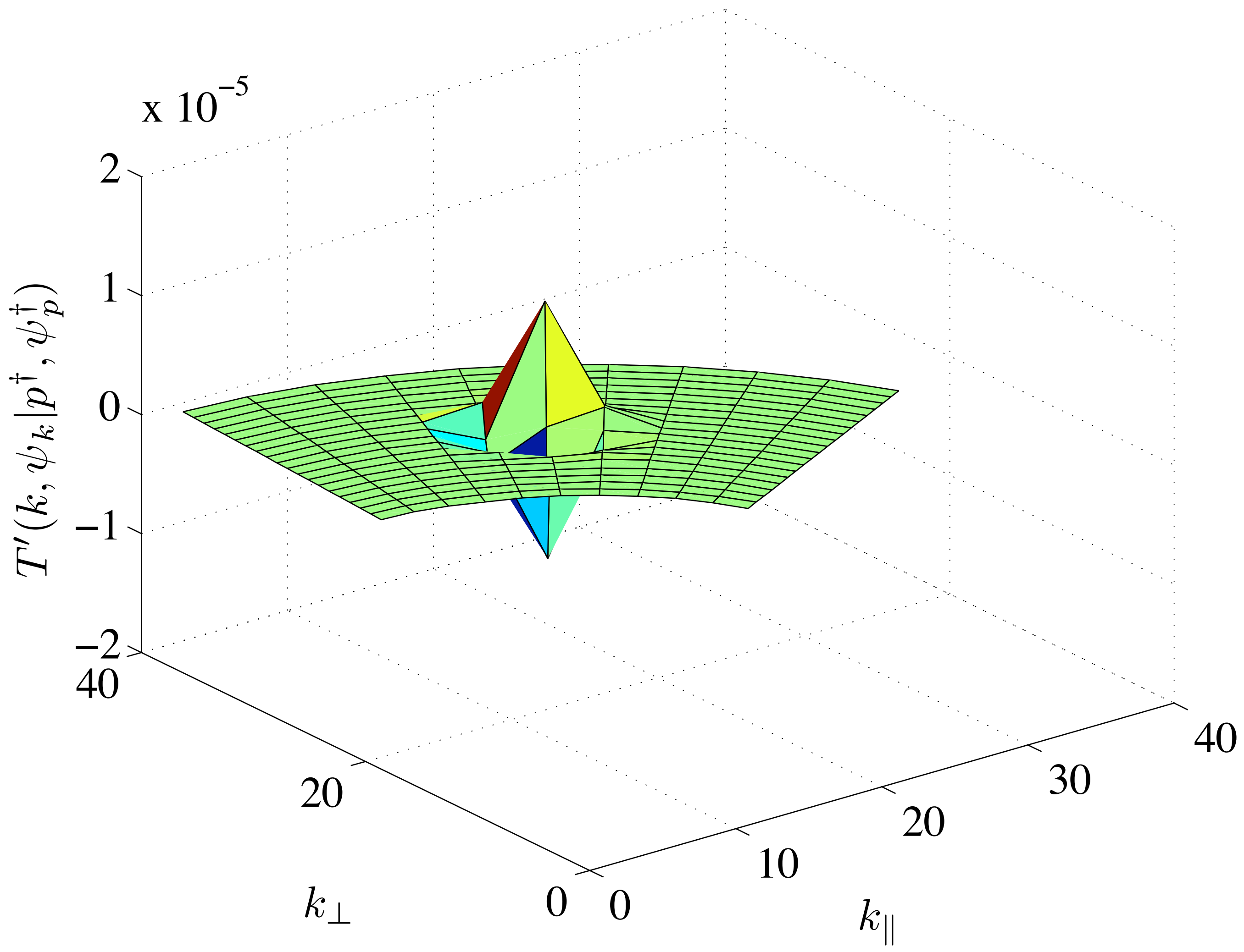


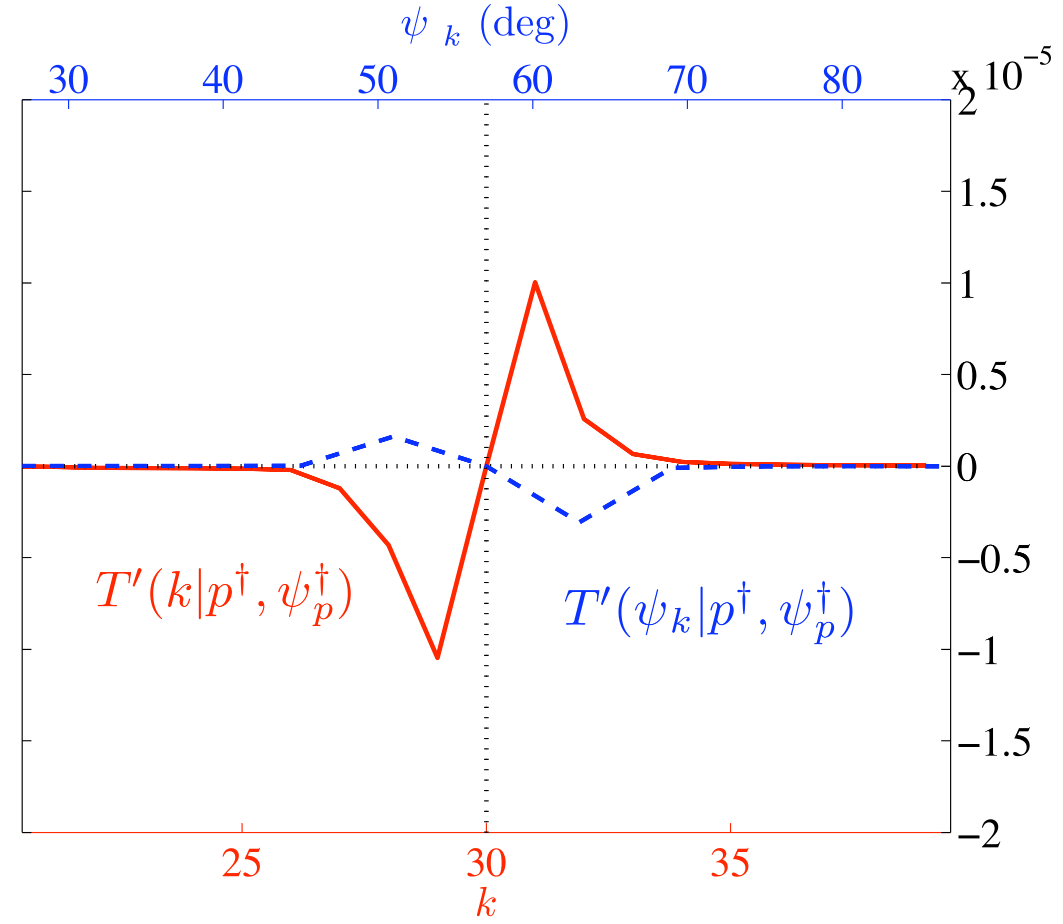
Ring to ring transfers (II)
N=0





Ring to ring transfers (III)
N=3





Conclusions

- DNS of homogeneous MHD turbulence.
- 2D distribution of energy and transfer spectra.
- In MHD the transfer is not only radial but also angular.
- The angular and radial transfer have the same order of magnitude.