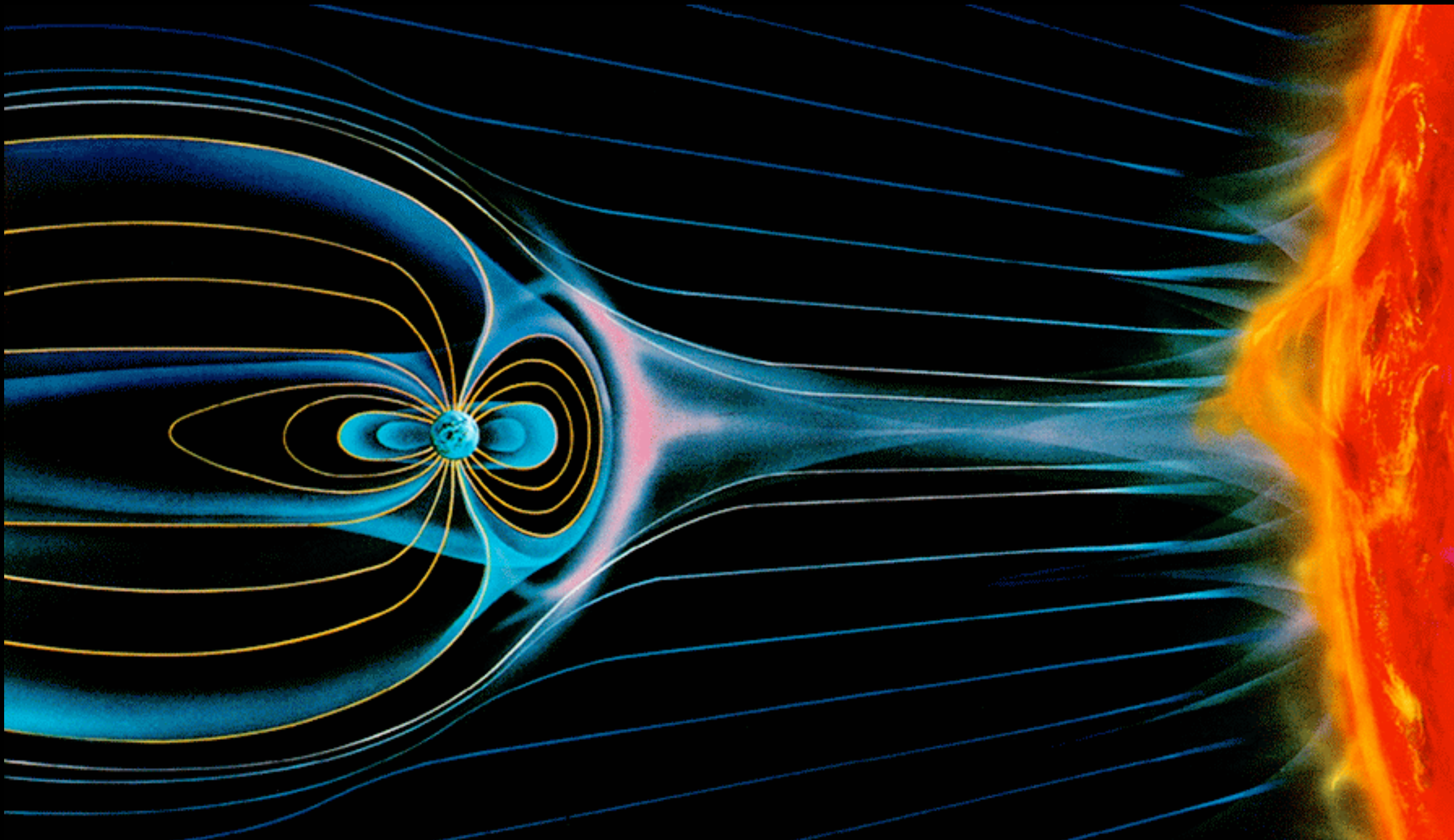
Hall MHD turbulence in the solar wind



Sébastien **GALTIER** (IAS / Univ. Paris-Sud)

&

E. Buchlin (Imperial College)

Solar Wind

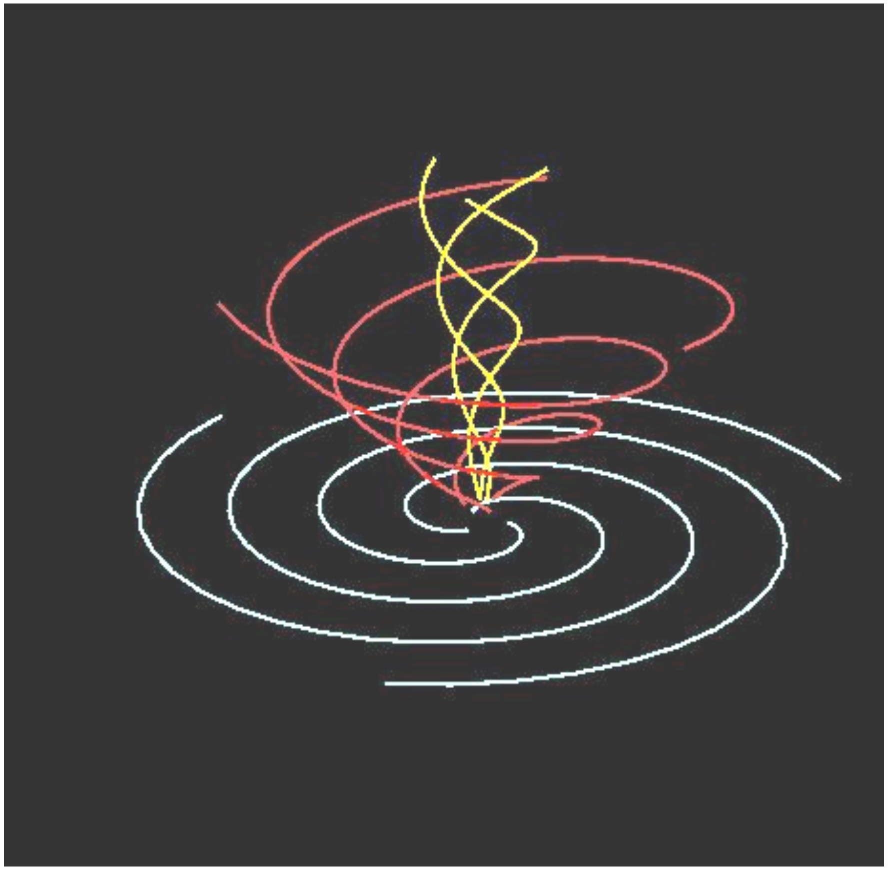
- Continual and variable outflow from the Sun (heliosphere ~ 100AU)

- Magnetized and collisionless plasma

- **Fast** and **slow** winds (> $20R_{SUN}$)

- Weak density variation (few %)

- Reynolds > 10^8

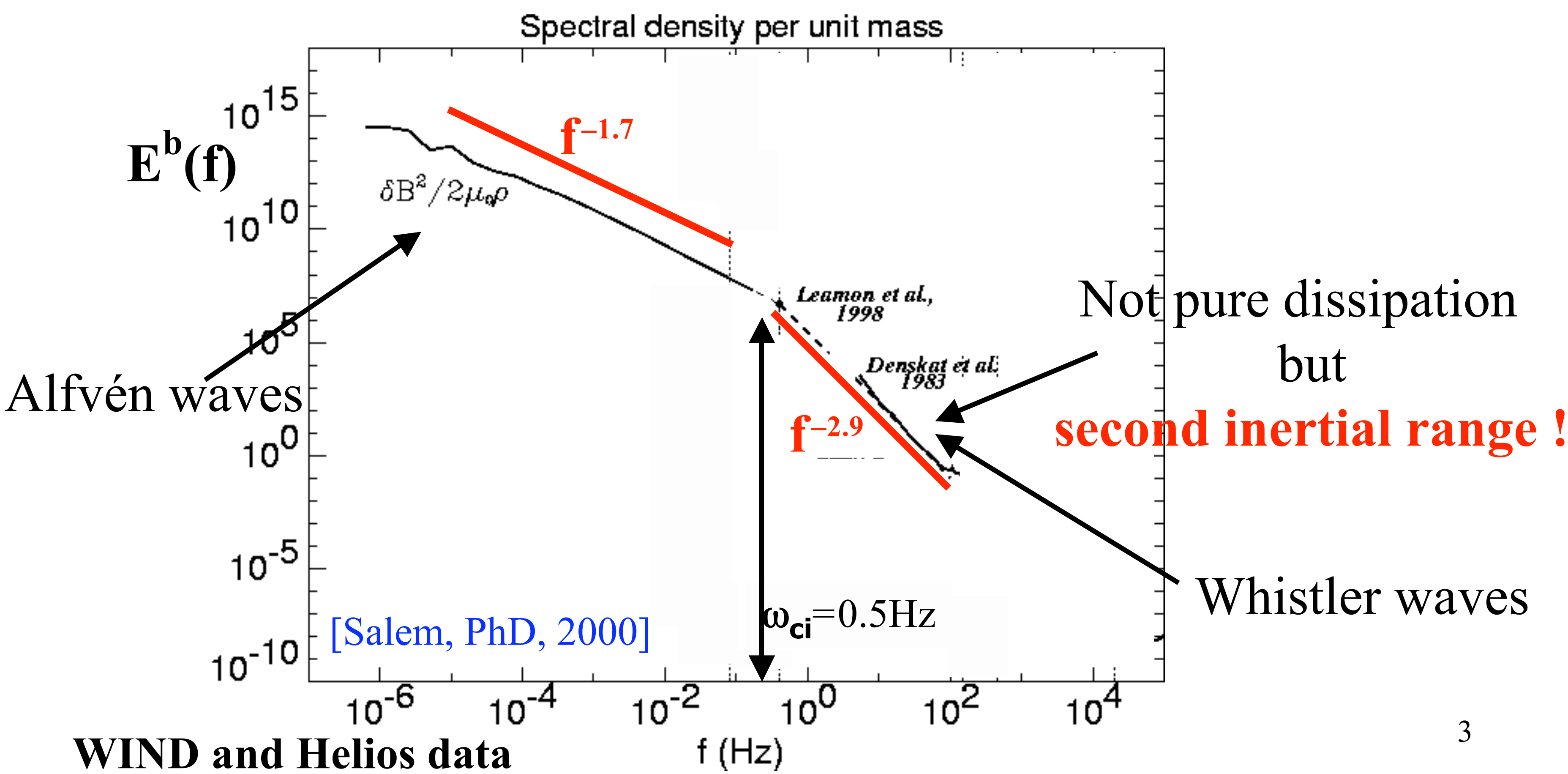


(E. Parker, 1958)

Magnetic field fluctuations

Steepening of the magnetic fluctuation spectra

[Coroniti et al., 1982; Denskat et al., 1983; Leamon et al., 1999; Smith et al., 2006]



Incompressible Hall MHD

Inviscid equations :

$$\nabla . \mathbf{v} = 0$$

$$\partial_t \mathbf{v} + \mathbf{v} . \nabla \mathbf{v} = -\nabla P_* + \mathbf{B} . \nabla \mathbf{B}$$

$$\partial_t \mathbf{B} + \mathbf{v} . \nabla \mathbf{B} = \mathbf{B} . \nabla \mathbf{v} - \mathbf{d_i} \nabla \mathbf{x} [(\nabla \mathbf{x} \mathbf{B}) \mathbf{x} \mathbf{B}]$$

$$\nabla . \mathbf{B} = 0$$

Ion inertial length : $\mathbf{d_i = B_o / \omega_{ci}}$ (d_i ~ 100 km at 1 AU)

1. Wave turbulence in Hall MHD

[Galtier, J. Plasma Physics **72**, 721-769, 2006]

We shall describe small scales in the inner solar wind

- We introduce : $\mathbf{B}(\mathbf{x},t) = B_0 \, \mathbf{e}_{//} + \varepsilon \mathbf{b}(\mathbf{x},t)$ with $0 < \varepsilon \ll 1$

  but $\mathbf{B_0}$ is in a **fixed** direction

- We develop perturbatively (in Fourier) the Hall MHD equations

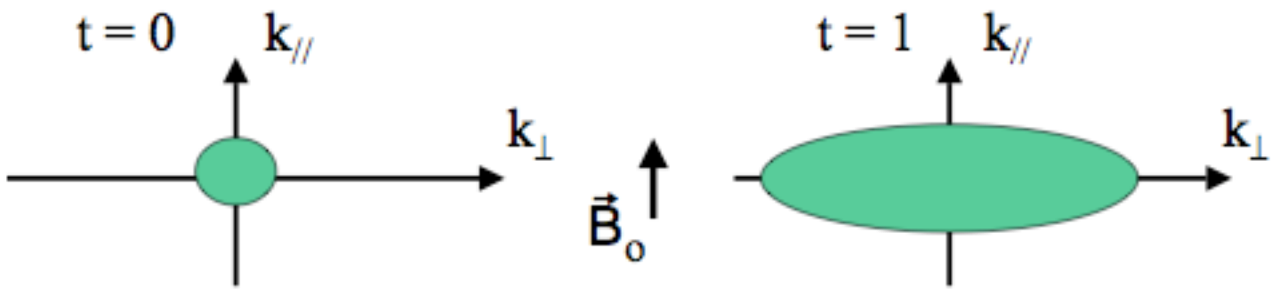
  (generalized Elsässer variables)

- We derive the asymptotically **exact** 3D wave kinetic equations

  $\tau_{nl} \gg \tau_w$ [Zakharov et al., 1992; Newell et al., 2001]

Wave HMHD turbulence properties

- Global tendency (at any scales) towards spectral **anisotropy** :



- The master equations are :

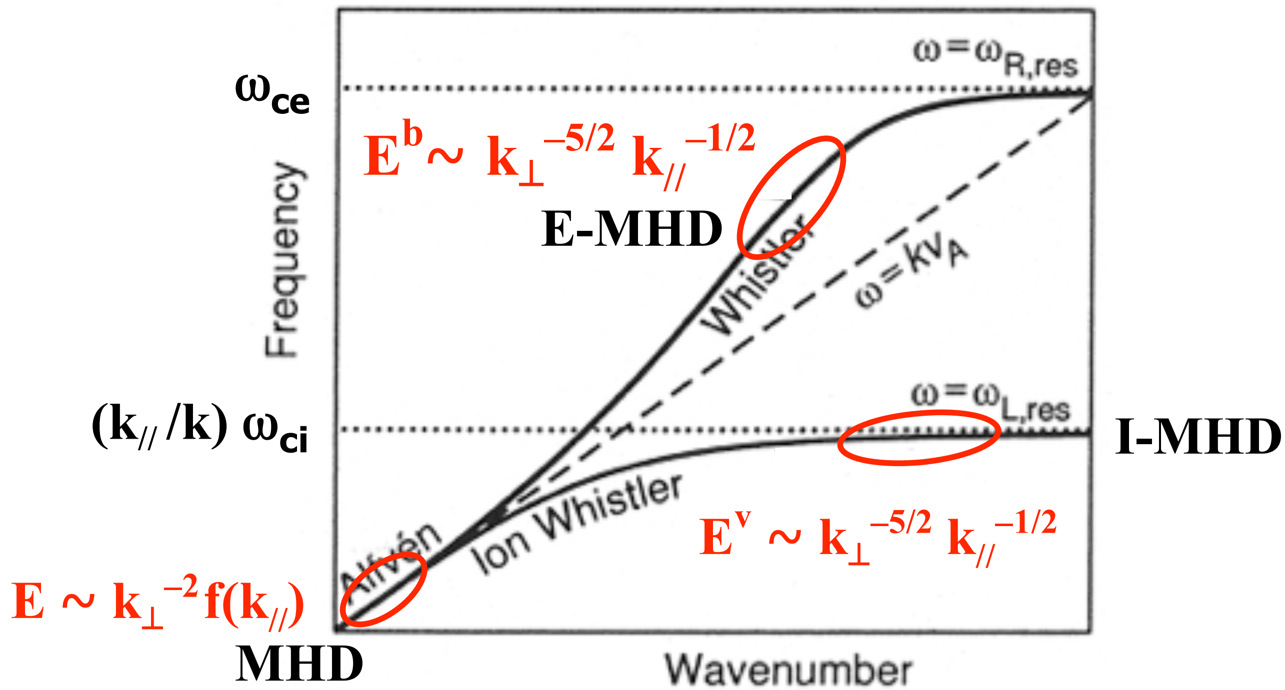
$$\partial_t \begin{Bmatrix} E^V(\mathbf{k}) \\ E^B(\mathbf{k}) \end{Bmatrix} = \frac{\pi \epsilon^2}{8\, d_i^2 B_0^2} \int \sum_{\substack{\Lambda, \Lambda_p, \Lambda_q \\ s, s_p, s_q}} \left(\frac{\sin \psi_k}{k}\right)^2 \frac{(\Lambda k + \Lambda_p p + \Lambda_q q)^2 \left(1 - \xi_\Lambda^{-s^2} \xi_{\Lambda_p}^{-s_p{}^2} \xi_{\Lambda_q}^{-s_q{}^2}\right)^2}{(1 + \xi_\Lambda^{-s^2})(1 + \xi_{\Lambda_p}^{-s_p{}^2})(1 + \xi_{\Lambda_q}^{-s_q{}^2})}$$

$$\left(\frac{\xi_{\Lambda_q}^{s_q} - \xi_{\Lambda_p}^{s_p}}{k_\perp}\right)^2 \begin{Bmatrix} \xi_\Lambda^{-s^2} \\ 1 \end{Bmatrix} \frac{\omega_\Lambda^s \omega_{\Lambda_p}^{s_p}}{\xi_\Lambda^{-s^2} + 1} \left(\frac{\xi_{\Lambda_q}^{-s_q{}^2} E^V(\mathbf{q}) - E^B(\mathbf{q})}{\xi_{\Lambda_q}^{-s_q{}^2} - 1}\right)$$

$$\left[\left(\frac{\xi_{\Lambda_p}^{-s_p{}^2} E^V(\mathbf{p}) - E^B(\mathbf{p})}{\xi_{\Lambda_p}^{-s_p{}^2} - 1}\right) - \left(\frac{\xi_\Lambda^{-s^2} E^V(\mathbf{k}) - E^B(\mathbf{k})}{\xi_\Lambda^{-s^2} - 1}\right)\right] \delta(\Omega_{k,pq})\, \delta_{k,pq}\, d\mathbf{p}\, d\mathbf{q}\,.$$

Wave HMHD turbulence properties

• The exact power law solutions show a **steepening** at small scales :



→ Also explained by a simple anisotropic **phenomenology !**

2. Strong turbulence in Hall MHD

- What do we know about strong Hall MHD turbulence ? *Not a lot…*

➔ DNS are restricted to (very) low Reynolds numbers

➔ Difficulties to get any multi-scaling

[*eg*. Ghosh et al., 1996; Mininni et al., 2003-2006]

Wait some decades or modify the strategy !

Strong turbulence in Hall MHD

[Galtier & Buchlin, Astrophys. J. **656**, 2007]

Numerical investigation through a **3D cascade model** :

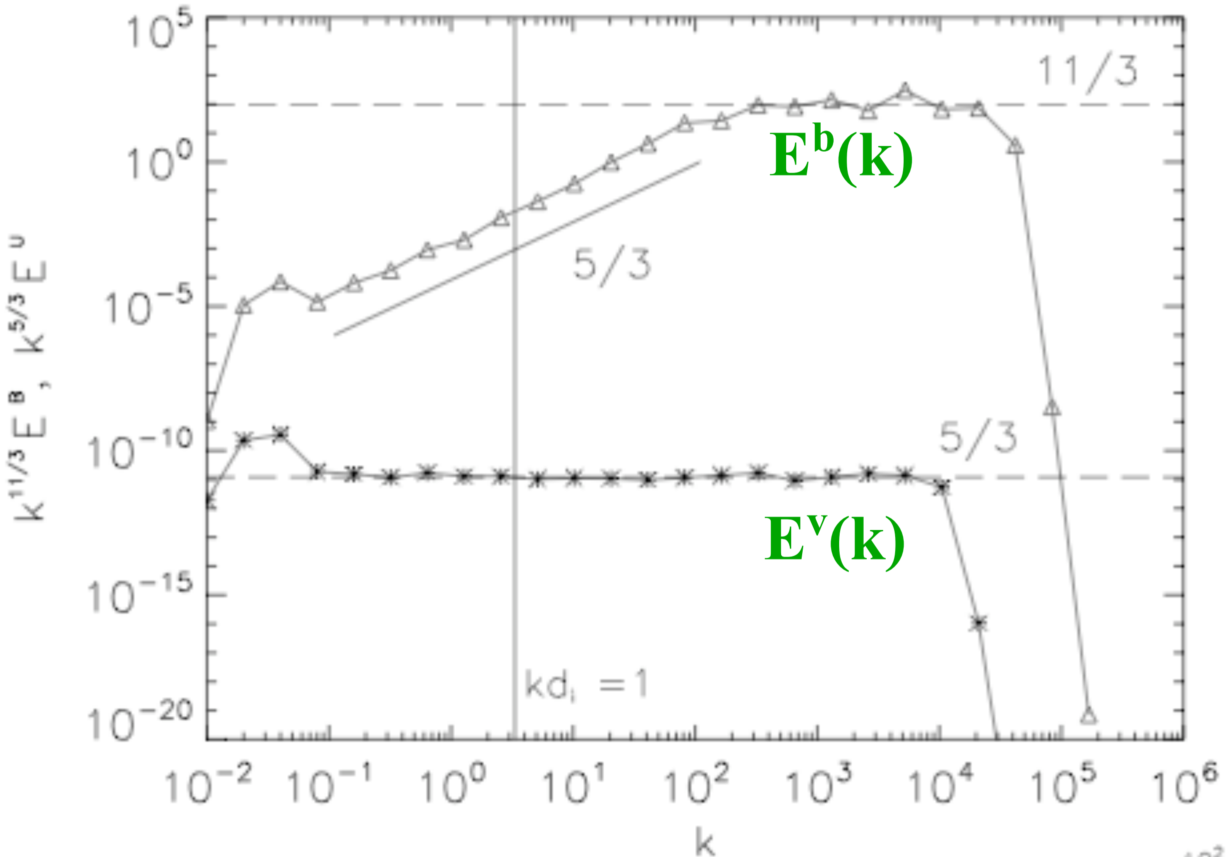
$$\frac{\partial V_n}{\partial t} + \nu_2 k_n^4 V_n =$$

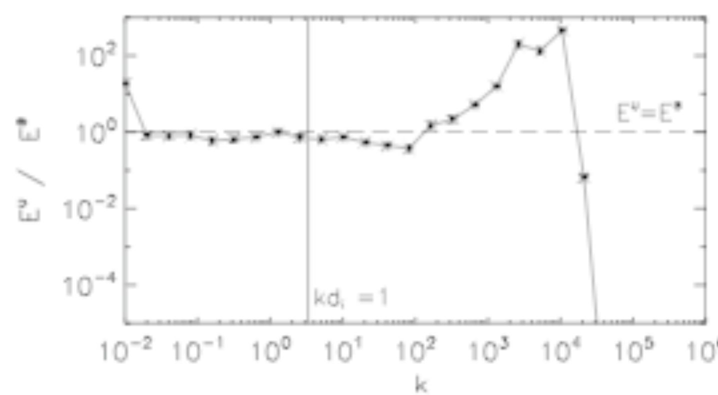
$$ik_n\left(V_{n+1}V_{n+2} - B_{n+1}B_{n+2} - \frac{V_{n-1}V_{n+1} - B_{n-1}B_{n+1}}{4} - \frac{V_{n-2}V_{n-1} - B_{n-2}B_{n-1}}{8}\right)^*,$$

$$\frac{\partial B_n}{\partial t} + \eta_2 k_n^4 B_n =$$

$$\frac{ik_n}{6}\left[(V_{n+1}B_{n+2} - B_{n+1}V_{n+2}) + (V_{n-1}B_{n+1} - B_{n-1}V_{n+1}) + (V_{n-2}B_{n-1} - B_{n-2}V_{n-1})\right]^*$$
$$+ (-1)^n i d_i k_n^2\left(B_{n+1}B_{n+2} - \frac{B_{n-1}B_{n+1}}{4} - \frac{B_{n-2}B_{n-1}}{8}\right)^*$$

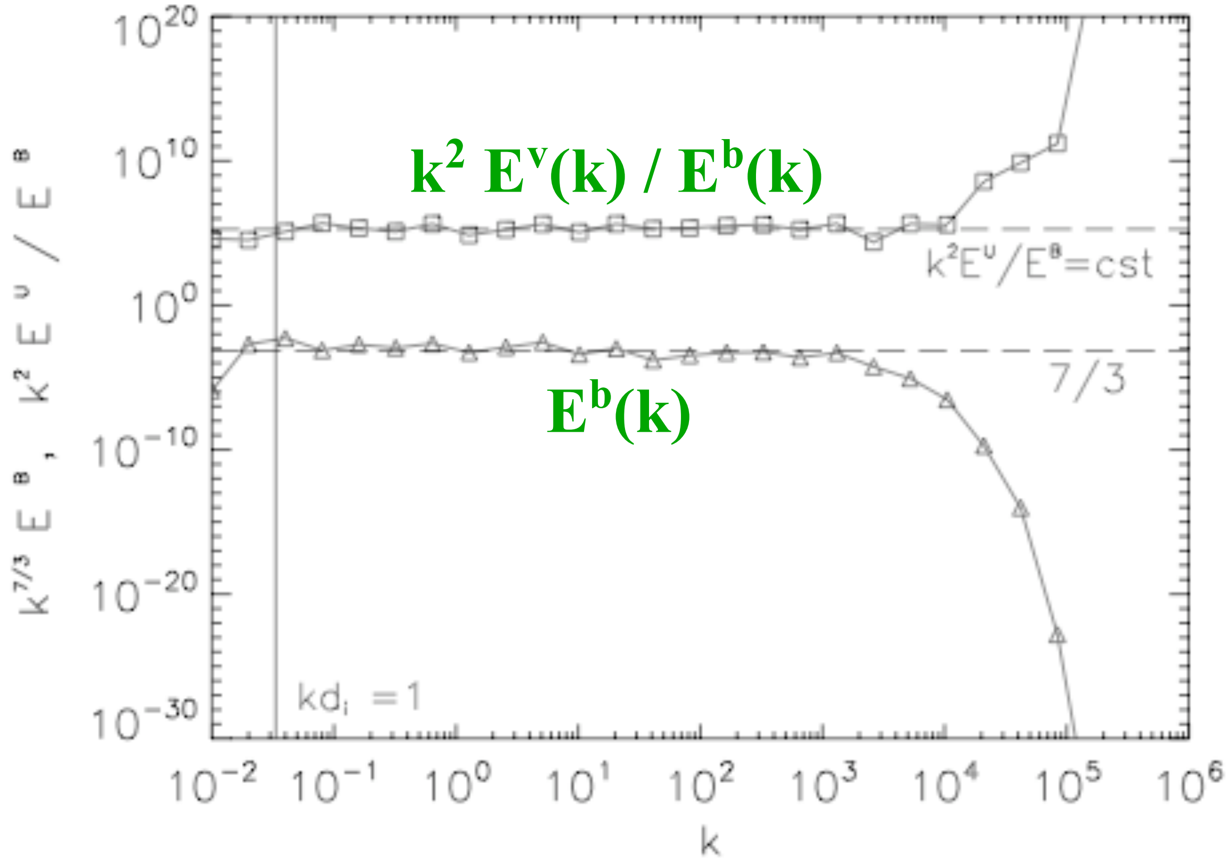
Strong turbulence in Hall MHD





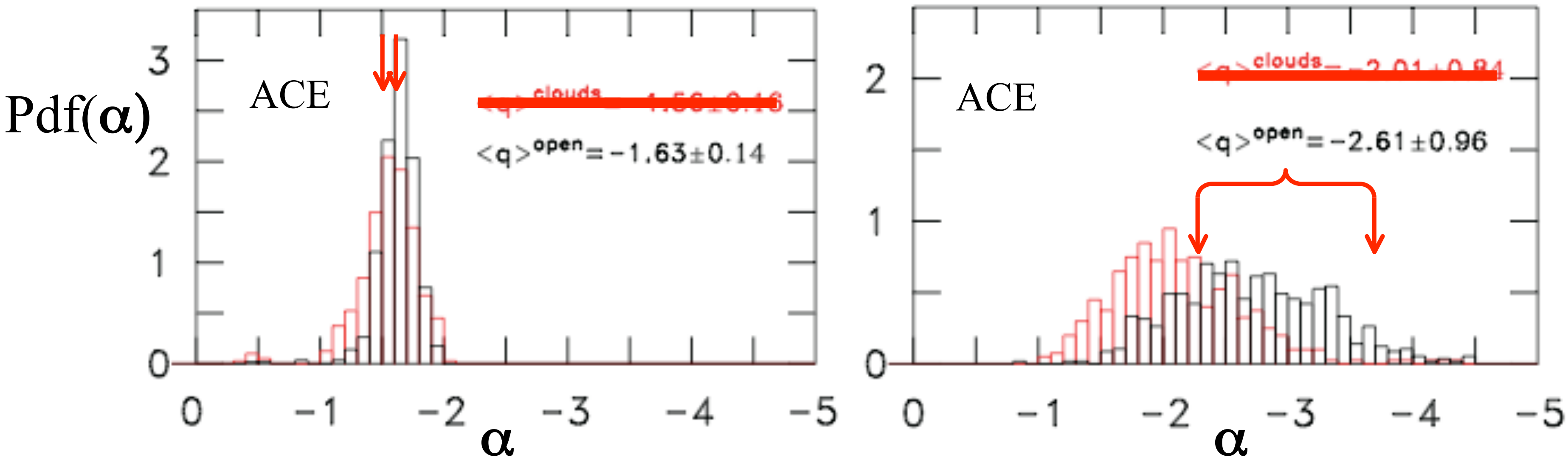
Non trivial steepening !

Strong turbulence in Hall MHD



Magnetic fluctuation power law spectrum

- Latest observations : [Smith et al., 2006]



- Strong Hall MHD turbulence : $E^b(k) \sim k^{-\alpha}$, where α = **7/3** → **11/3**

→ Physically the value may depend on the **cyclotron absorption**

Conclusion

- **First** « strong » results in Hall MHD turbulence

- **Hall MHD** turbulence is a relevant solar wind model

  → Weak **and** strong turbulence lead to a **steepening**

- Role of **asymmetric** wave flux, Taylor hypothesis…?

  (f → wavevector **k**)

  → Crucial for **many** problems in astrophysics