Turbulence Budgets in the Wind Flow Over Homogeneous Forests

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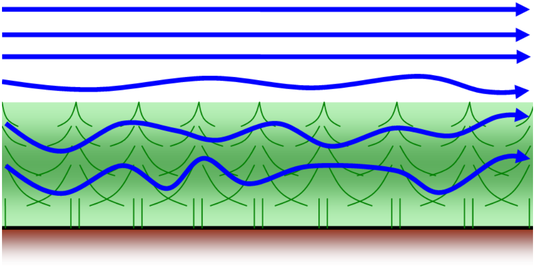
Introduction

The presence of plant canopy on the surrounding terrain of a wind farm is an important issue in the modelling of its behaviour.

Aventos wind flow prediction simulations must include forest canopy models to adequately analyse those locations.

Objectives

Simulation of a flat and continuous ideal forest.



- Check the quality and reliability of the existing RaNS canopy models.
- Understand the phenomena involved in a flow among and above a forested region.

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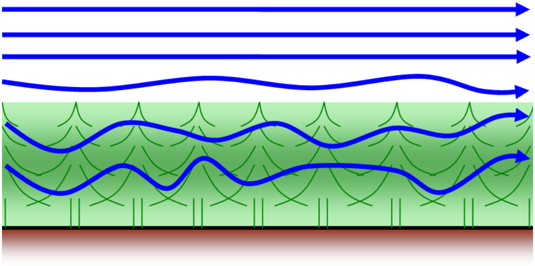
Simulation of a flat and continuous ideal forest.



There are obvious difficulties on find field data.
As a result, some LES was carried out, based on Shaw and Schumann (1992).

LES

LES equations:

$$\begin{aligned}\frac{\partial \overline{u}_j}{\partial x_j} &= 0,\\ \rho\frac{\partial \overline{u}_i}{\partial t} + \rho\frac{\partial\left(\overline{u}_j\overline{u}_i\right)}{\partial x_j} &= -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu\frac{\partial \overline{u}_i}{\partial x_j}\right) + \frac{\partial \tau_{ij}}{\partial x_j} + F_i.\end{aligned}$$

τ_{ij} was calculated using the dynamic model (Germano, 1991) with a Lagrangian approach (Meneveau, 1996) for averaging along the fluid particle trajectories.

Canopy drag force:

$$F_i = -C_D a(z)\rho\left|\overline{\mathbf{u}}\right|\ \overline{u}_i$$

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RaNS k-ε Turbulent Model

k-ε turbulent model equations:

$$\begin{aligned} \frac{\partial U_j}{\partial x_j} &= 0, \\ \rho \frac{\partial U_i}{\partial t} + \rho \frac{\partial (U_i U_j)}{\partial x_j} &= -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} (\sigma_{ij} + \tau_{ij}) + F_i, \end{aligned}$$

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k and ε equations:

$$\begin{aligned} \rho \frac{\partial k}{\partial t} + \rho \frac{\partial (U_i k)}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \mathcal{P}_k - \rho\varepsilon + S_k, \\ \rho \frac{\partial \varepsilon}{\partial t} + \rho \frac{\partial (U_i \varepsilon)}{\partial x_j} &= \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + \frac{\varepsilon}{k} (C_{\varepsilon 1} \mathcal{P}_k - C_{\varepsilon 2} \rho \varepsilon) + S_\varepsilon. \end{aligned}$$

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RaNS k-ε Turbulent Model

Canopy k and ε source terms

General formulation (Katul, 2004):

$$
\begin{aligned}
S_k &= \rho C_z \left(\beta_p \left| \mathbf{U} \right|^3 - \beta_d \left| \mathbf{U} \right| k \right), \\
S_\varepsilon &= \rho C_z \left(C_{\varepsilon 4} \beta_p \frac{\varepsilon}{k} \left| \mathbf{U} \right|^3 - C_{\varepsilon 5} \beta_d \left| \mathbf{U} \right| \varepsilon \right).
\end{aligned}
$$

Constants set used in the different canopy models:

	β_p	β_d	$C_{\varepsilon 4}$	$C_{\varepsilon 5}$
Svensson and Häggkvist, 1990	1.0	0.0	1.95	0.0
Green, 1992	1.0	4.0	1.5	1.5
Liu et al., 1996	1.0	4.0	1.5	0.6
Sanz, 2003 / Katul et al., 2004	1.0	5.1	0.9	0.9

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Numerical Implementation

LES

Domain of
192 m $\times$ 96 m $\times$ 60 m
Mesh:

- 192 $\times$ 96 $\times$ 60
- Nodes equally spaced in all directions.

Imposed Courant number
$$\frac{|\overline{\mathbf{u}}|_{max}\,\Delta t}{\Delta x} = 0.2.$$

Simulation time: 6000 s.

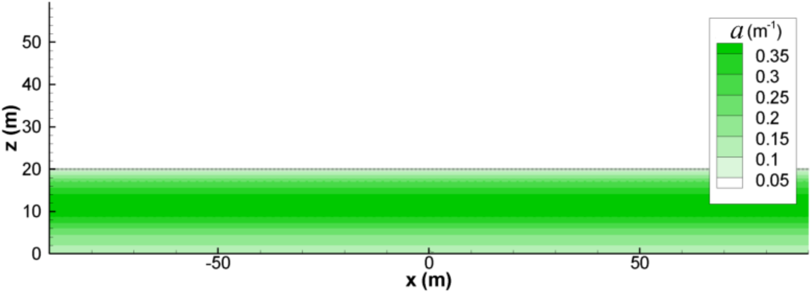
RaNS k-ε

Domain of 40 m $\times$ 10 m $\times$ 60 m
Mesh:

- 20 $\times$ 5 $\times$ 60
- Nodes equally spaced in horizontal directions.
- Vertical mesh from 0.1 m near the ground till 2.0 m at the top of the domain.

Numerical Implementation

- Flat and continuous ideal forest; variable leaf area density.
- Longitudinal and lateral periodic boundary conditions; symmetry condition at the top of the domain.
- Longitudinal mean velocity U_{mean} =2 m/s imposed.



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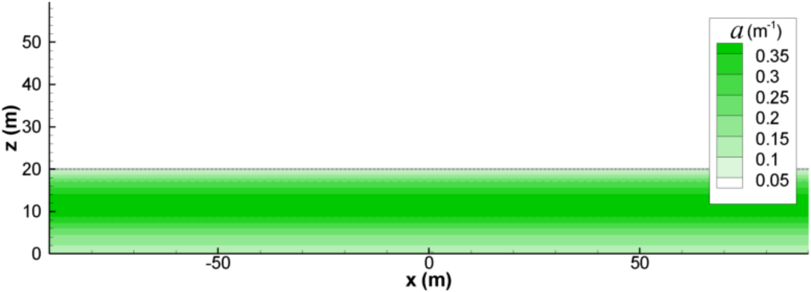

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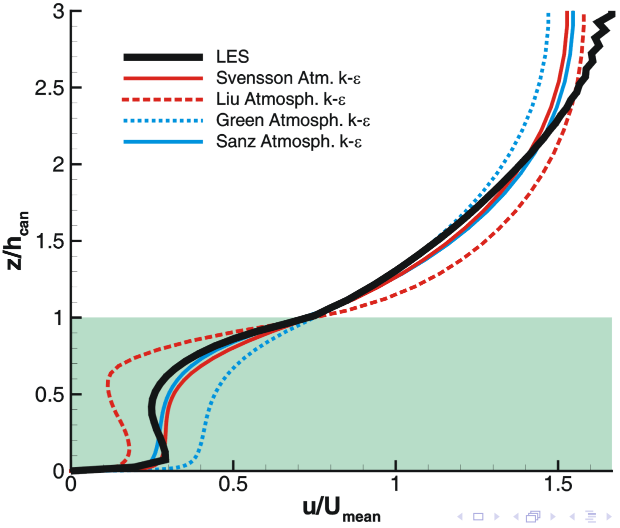
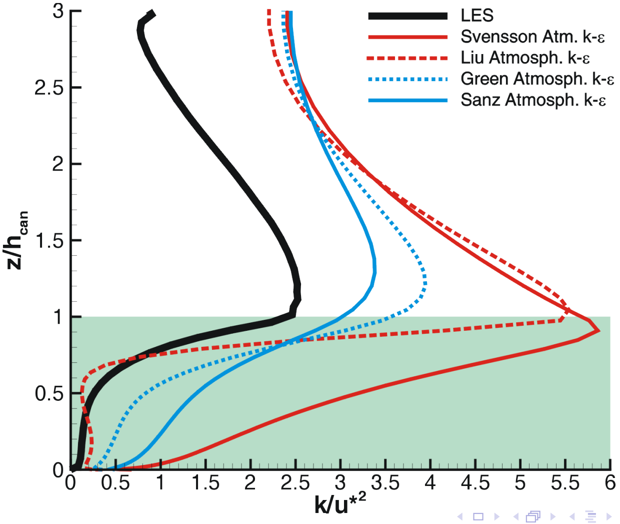
LES

Velocity


LES
Svensson Atm. k-ε
Liu Atmosph. k-ε
Green Atmosph. k-ε
Sanz Atmosph. k-ε
z/h_can
u/U_mean
0
0.5
1
1.5
2
2.5
3
0
0.5
1
1.5

Turbulent Kinetic Energy - k



u^* - Friction velocity at canopy top.

TKE Budget Terms

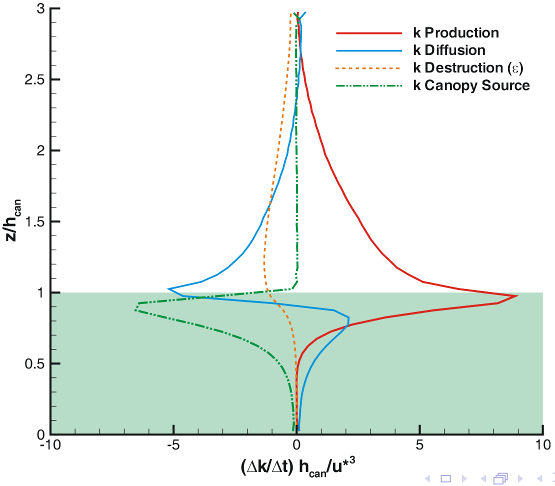
k equation

$$\rho\frac{\partial k}{\partial t} + \rho\frac{\partial\left(U_i k\right)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + \mathcal{P}_k - \rho\varepsilon + S_k$$

- Production
- Diffusion
- ε - Destruction
- Canopy Source

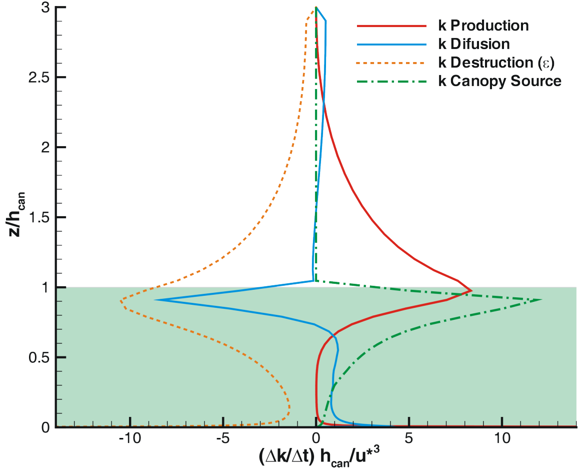
TKE Budget Terms

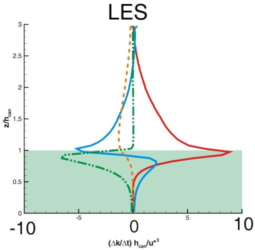
LES



TKE Budget Terms

Svensson, 1990





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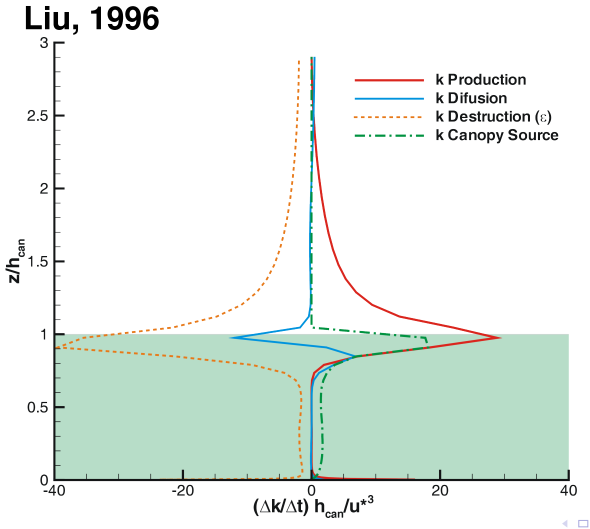

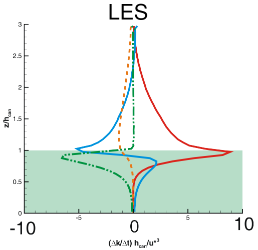

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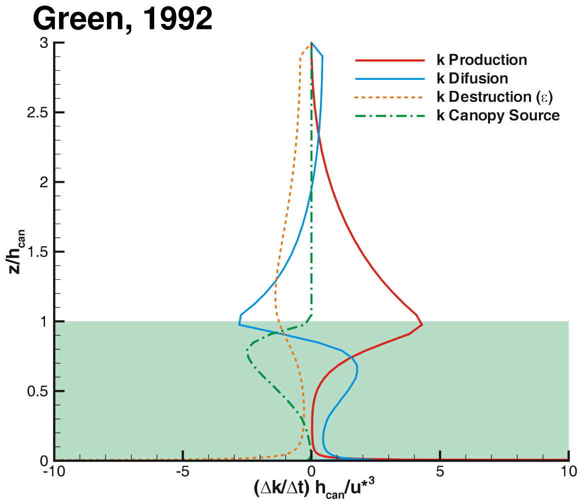

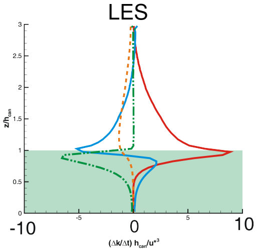

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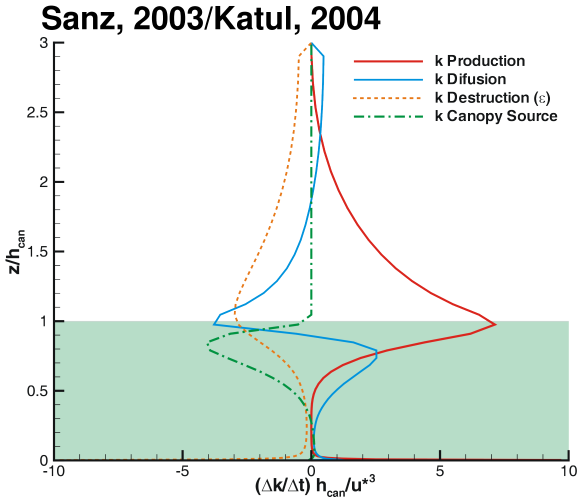

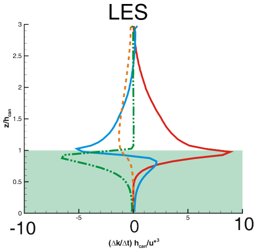

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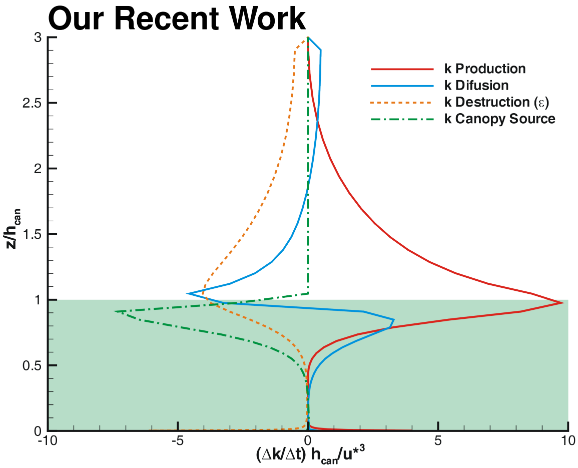

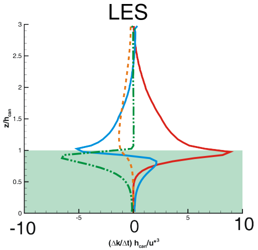

Conclusions

- Most of RANS canopy models produce acceptable velocity results.
- TKE results are more sensible to RANS canopy models.
- Two of the four RANS canopy models fail to mimic the physics of turbulence.
- Sanz (2003)/Katul (2004) canopy model is the most correct and balanced canopy model.

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