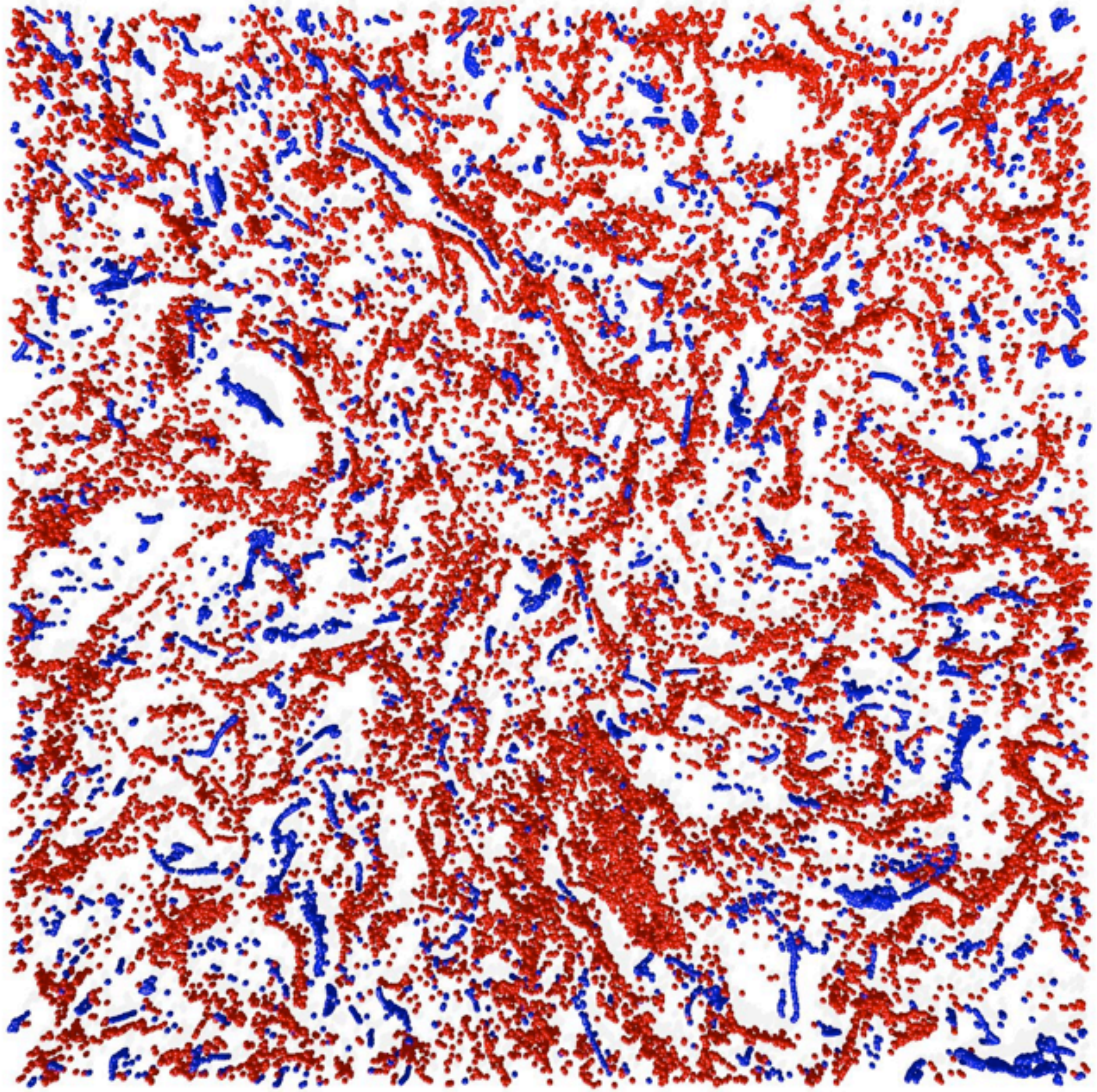
Concentration and segregation of particles and bubbles by turbulence : a numerical investigation



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Phenomenology of particles in turbulence

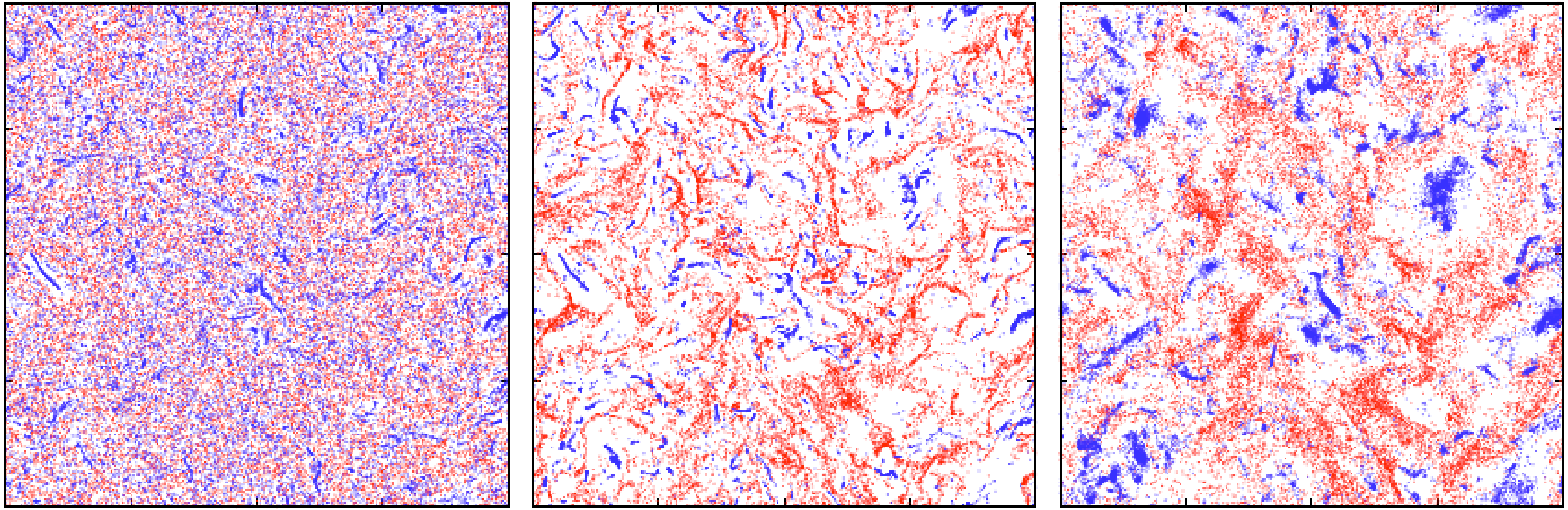
1) Ejection/injection of heavy/light particles from/in vortices

preferential concentrations Maxey (1987), Squires & Eaton (1991), Fessler Eaton (1994)

2) Finite response time to fluid fluctuations (smoothing of fast time scales)

Slice ~512x512x8 η from DNS

heavy particles $\rho_p >> \rho_f$ (red) and light particles $\rho_p << \rho_f$ (blue)



Stokes number St = 0.1 **St = 1** **St = 4.1**

Objective

study **statistical properties** and

correlation with the carrier flow structures

of inertial particle clusters

for a **wide range** of density ratios and response times

quantitative measure

of the effects of particle inertia

in turbulent flows.

Numerical Simulations

Incompressible flow field $\boldsymbol{u}$

Navier-Stokes

$$\mathrm{D}_t \boldsymbol{u} = \partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \Delta \boldsymbol{u} + \boldsymbol{F}$$

- Homogeneous isotropic turbulence
- Large scale forcing
- Periodic cubic domain
- $Re_\lambda \approx 78$ $(L^3 = 128^3)$
- $Re_\lambda \approx 180$ $(L^3 = 512^3)$

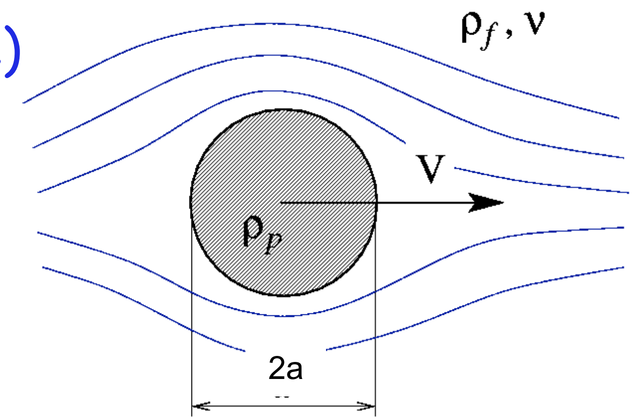
Particle's equation of motion

Particles with $a << \eta$ (Kolmogorov scale)

Particle Reynolds: $\mathrm{Re}_a = a\mathrm{v}/\nu << 1$

Dilute suspensions (no collisions),

no gravity



$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \beta \frac{D}{Dt}\mathbf{u}(\mathbf{x}(t),t) - \frac{1}{\tau_p}(\mathbf{v} - \mathbf{u}(\mathbf{x}(t),t))$$

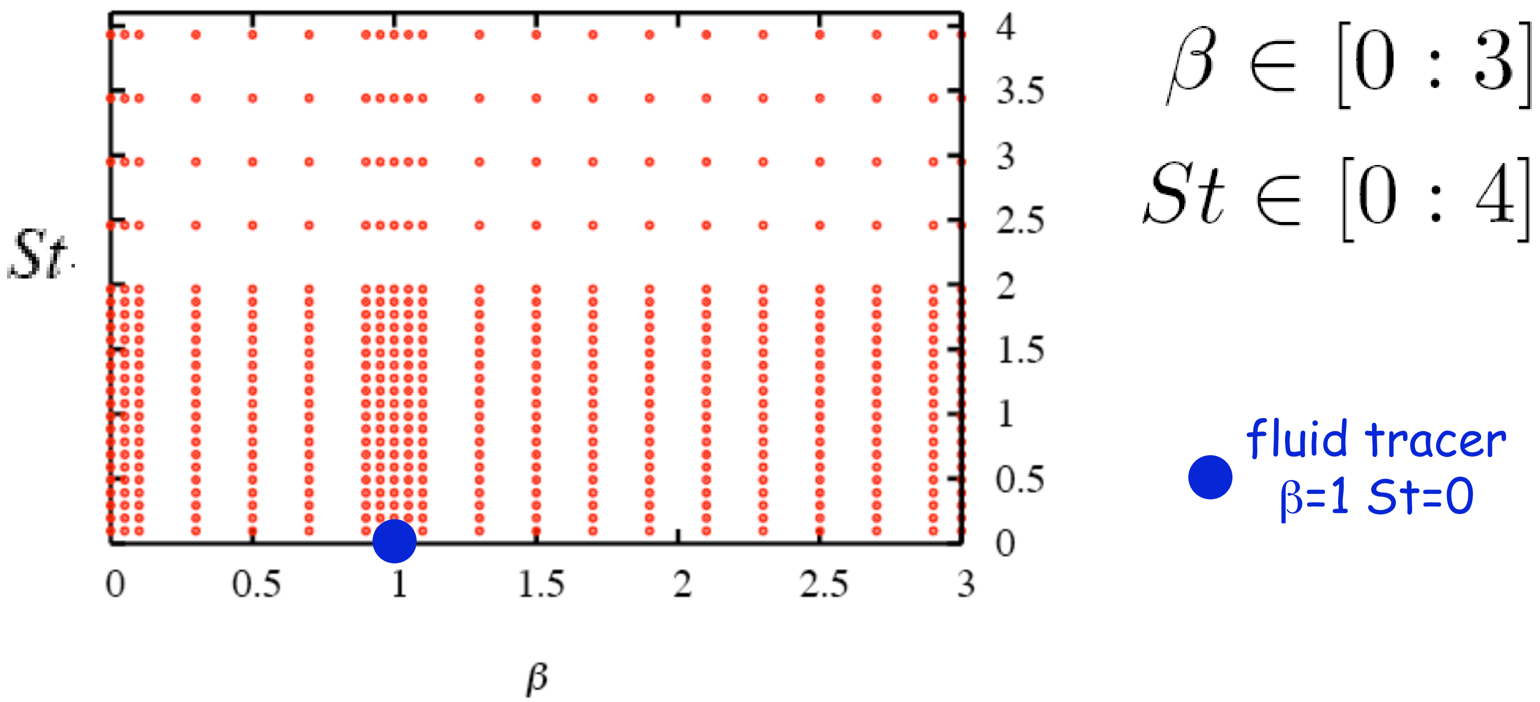
$$\boxed{\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}} \qquad \tau_p = \frac{1}{3\beta}\frac{a^2}{\nu} \qquad \boxed{St \equiv \frac{\tau_p}{\tau_\eta} = \frac{1}{3\beta}\frac{a^2}{\eta^2}}$$

(essentially: Maxey & Riley *Phys. Fluids* 1983, T.R.Auton et al. *JFM* 1988)

Numerical simulations: particles

Total number of particles: $N_{tot} \approx 50 \cdot 10^6$:

Grouped in ~500 types on the β-St parameter-space:



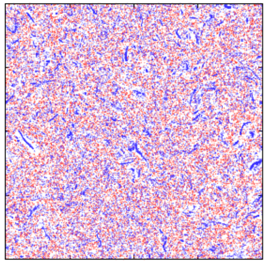
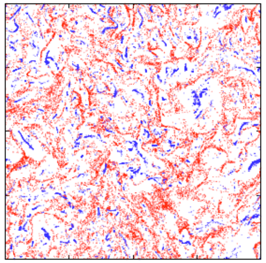
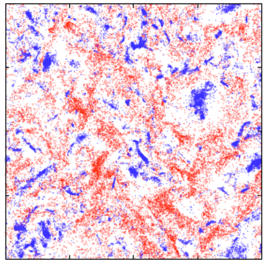
$$\beta \in [0:3]$$

$$St \in [0:4]$$

Particles tracked for 10s T_{eddy}

Database: **particle's position, velocity, acceleration, fluid velocity and gradients at particle position**

Outline

(i) Clusters dependence on β and St (small-scale features):

- attractor (Kaplan-Yorke) dimension D_{KY}
- correlation dimension D_2
- Minkowski functionals.

(ii) Clusters correlation to local flow properties

- particle concentration conditioned to local flow topology.

(i) Kaplan-Yorke dimension: D_{KY}

Particle equations of motion
defines a dissipative dynamical system

Attractor's dimension in the (x,v) space:
Kaplan Yorke dimension D_{KY}

$$\boldsymbol{D_{KY}} \equiv J - \frac{\lambda_1 + \cdots + \lambda_J}{\lambda_{J+1}} \qquad \begin{array}{l} \lambda_1 + \cdots + \lambda_J \geq 0 \\ \lambda_1 + \cdots + \lambda_{J+1} < 0 \end{array}$$

6 Lyapunov exponents computed by tracking

$$\mathbf{R}(t) \equiv (\delta\mathbf{x}(t), \delta\mathbf{v}(t))$$

$$\frac{d\mathbf{R}}{dt} = \mathcal{M}_t \mathbf{R}$$

$$\lambda_i = \lim_{T\to\infty} \gamma_i(T)$$

stretching rates

Standard ortho-normalization
Gram-Schmidt procedure adopted

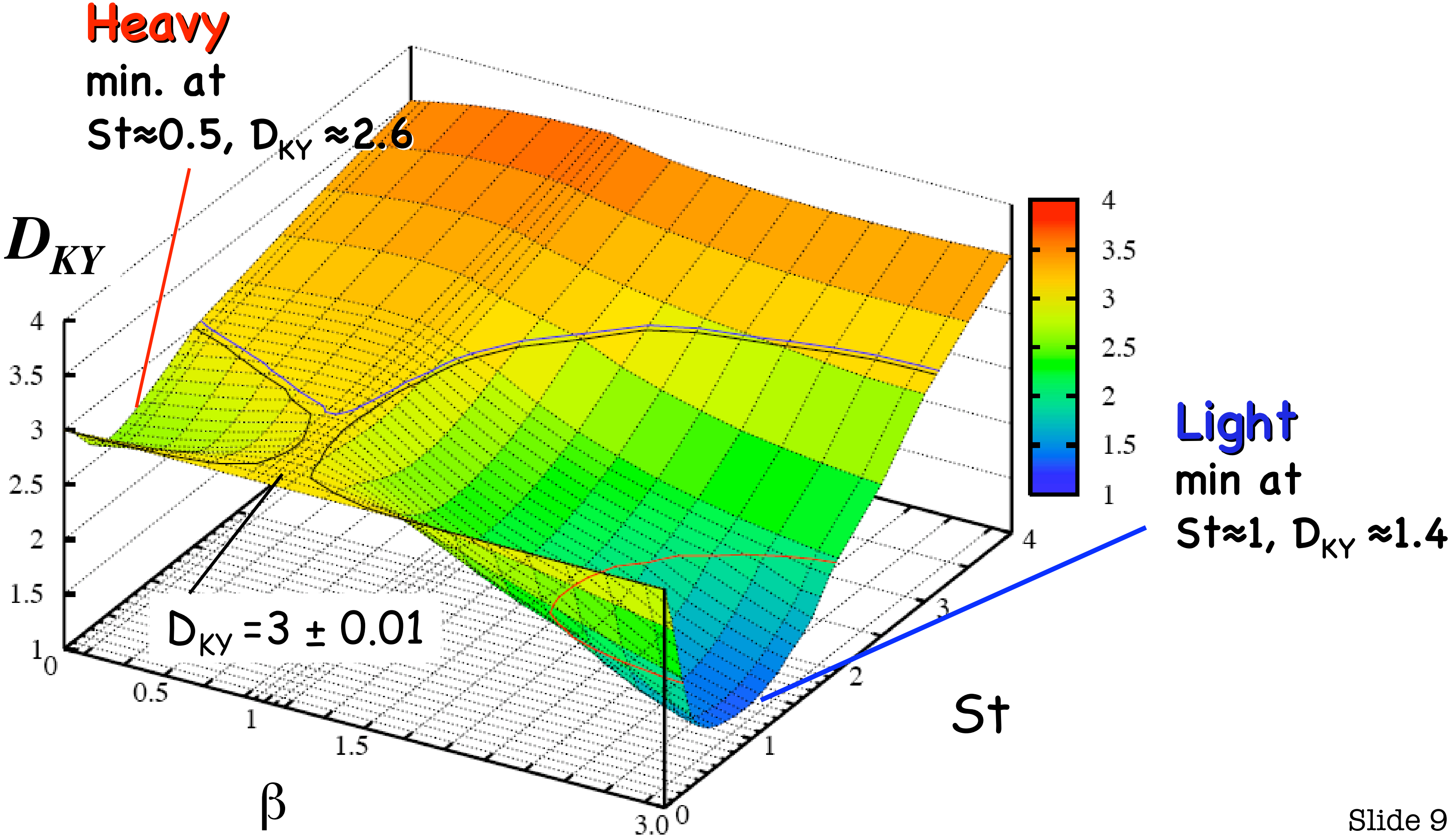
As in Bec *Phys. Fluids* (2003), Bec *JFM* (2005), Bec et al. *Phys. Fluids* (2006)

Kaplan-Yorke dimension

Balance between contraction and expansion

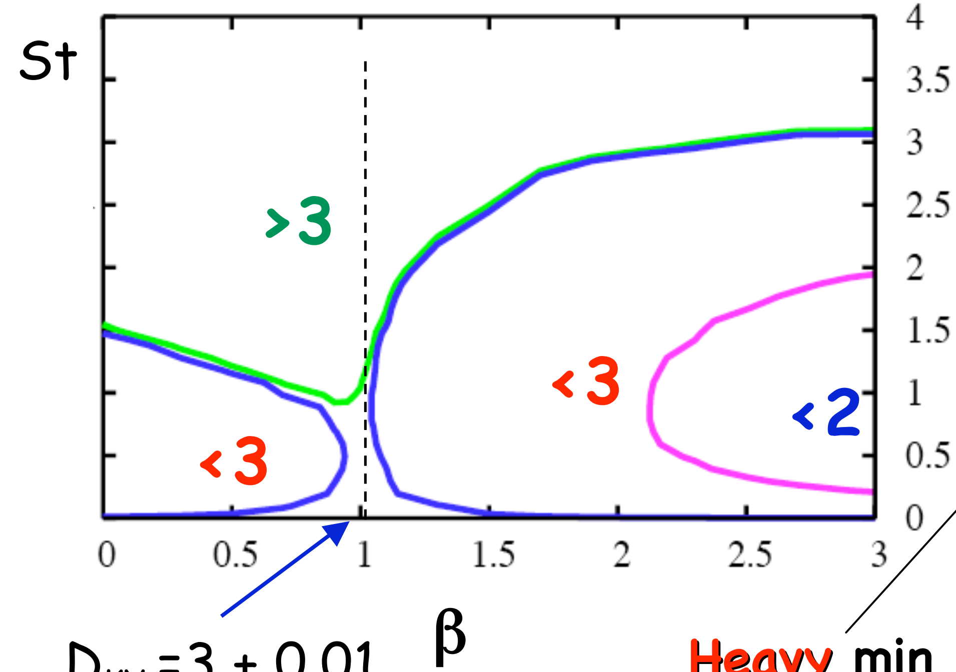
$$D_{KY} \equiv J - \frac{\lambda_1 + \cdots + \lambda_J}{\lambda_{J+1}}$$

$$\lambda_1 + \cdots + \lambda_J \geq 0$$
$$\lambda_1 + \cdots + \lambda_{J+1} < 0$$



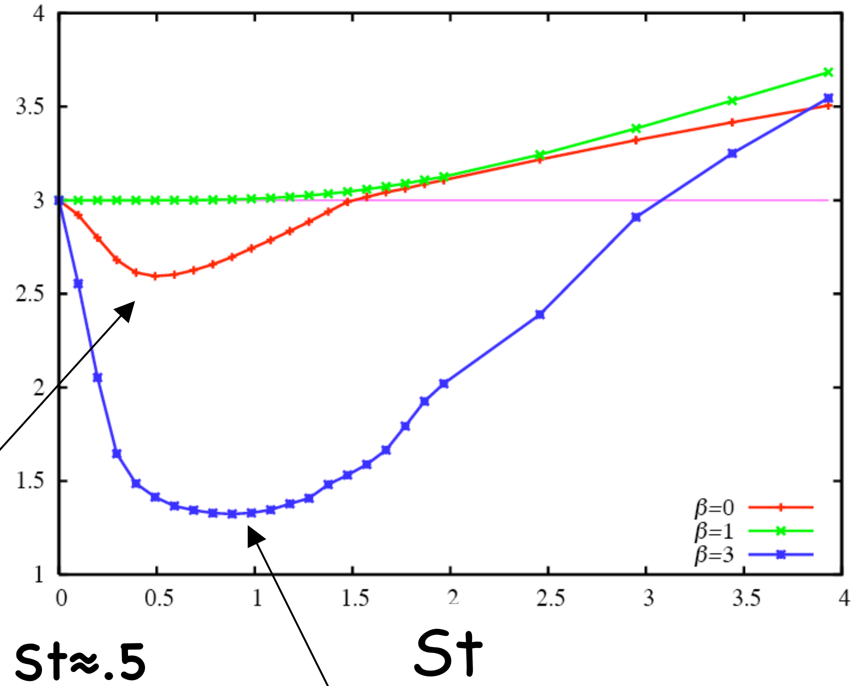
Projection of D_{KY}

Horizontal projection



$D_{KY} = 3 \pm 0.01$

Vertical projection



Heavy min at St≈.5

Light min at St≈1

Close to fractal dimension of
vortex filaments in turbulence
(Moisy&Jimenez JFM 04)

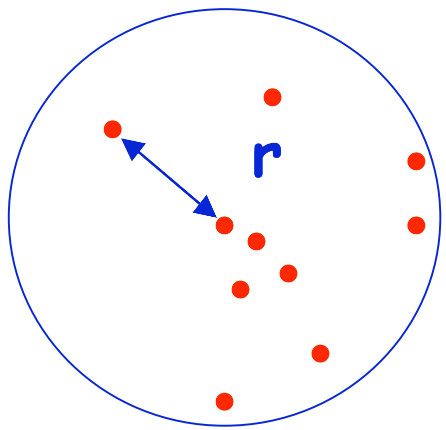
$$D_\omega \rightarrow 1.1 \pm 0.1$$

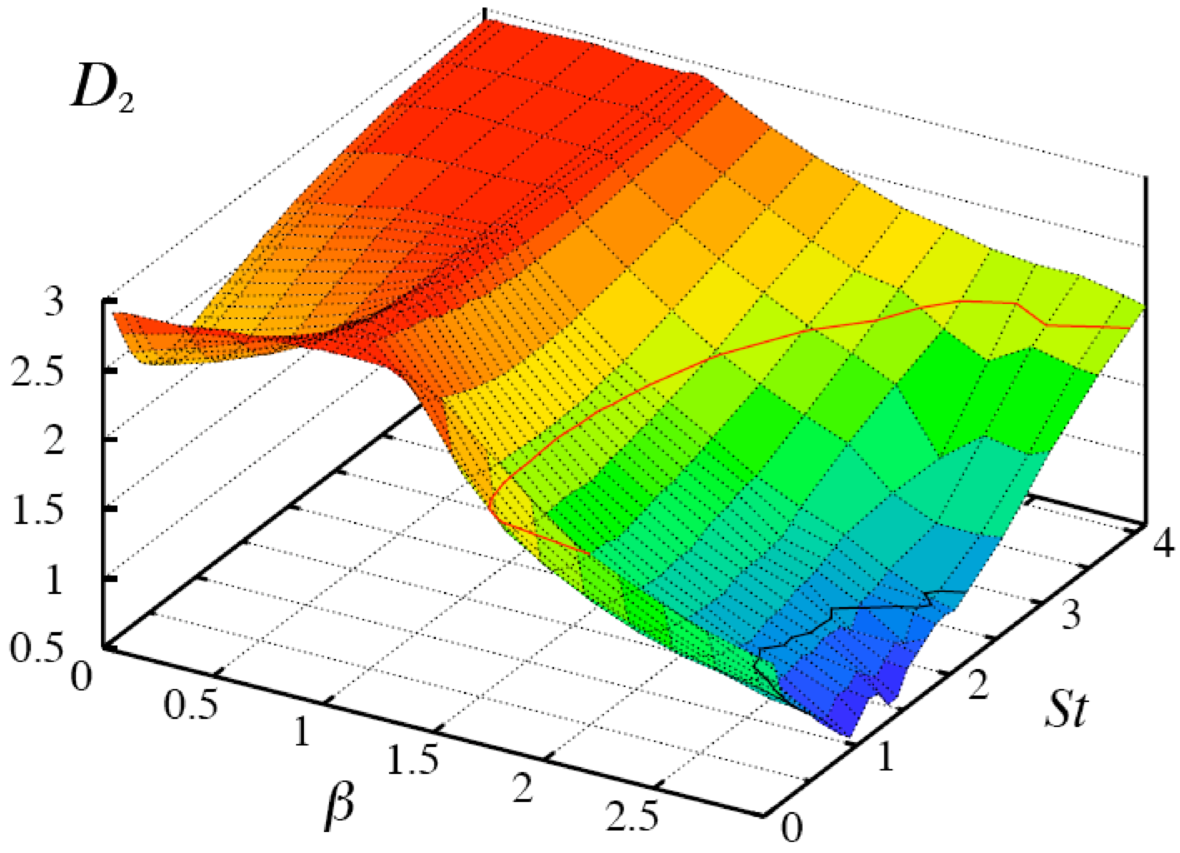
Correlation dimension D_2

$P_2(r)$ Probability to find a couple of particle whose distance is below r.

At $r \ll \eta$ $P_2(r) = A\, r^{\boxed{D_2}}$

fractal dimension hierarchy: $D_2 \leq D_1 = D_{KY}$





Same features as D_{KY}

More accessible in experiments

Morphological analysis of point clouds

- Put balls $\mathcal{B}_r(\mathbf{x}_i)$ with radius ***r*** around each particle ***i***

- Let ***r*** increase

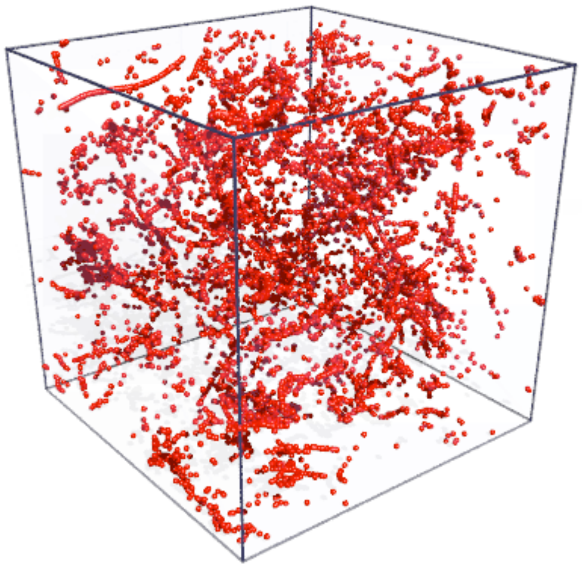
- Measure total **volume**, **surface**, **mean curvature** and **Euler characteristic** of the emerging structure

$$\mathcal{A}_r = \bigcup_{i=0}^{N} \mathcal{B}_r(\mathbf{x}_i)$$

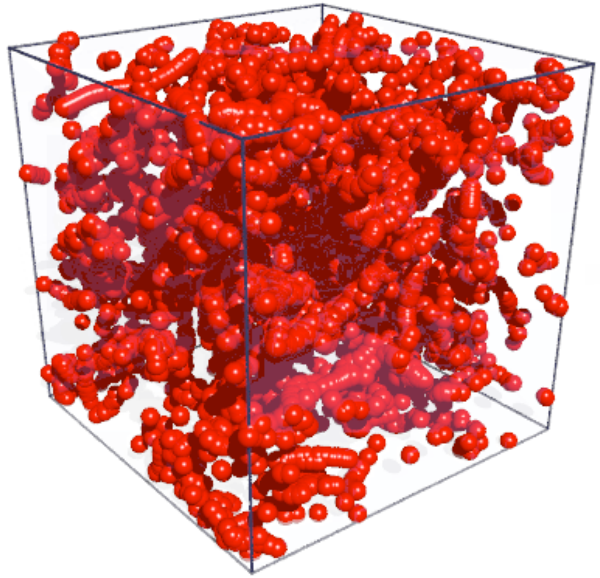
Minkowski functionals provide complete morphological characterization of point cloud!

Visualization of A_r

$2 \cdot 10^4$ particles with $\beta=3$ and St=1

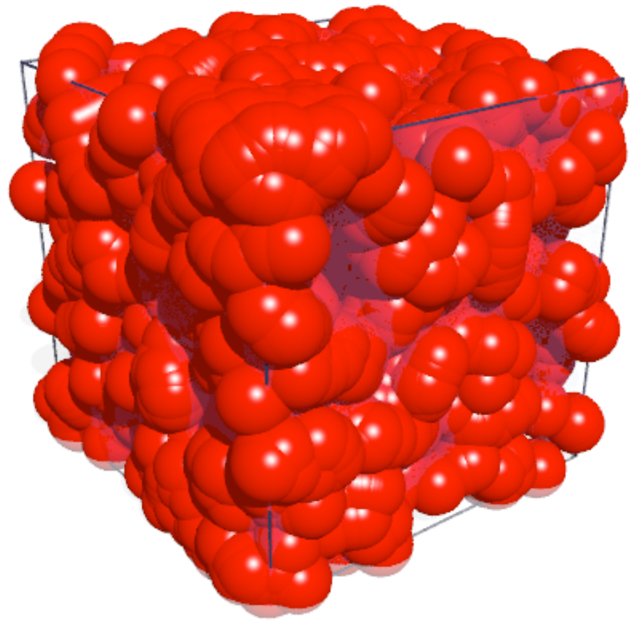


radius = 0.5 η



radius = 3 η

r



radius = 10 η

Minkowski functionals $V_\mu(r)$ in 3D

μ	$V_\mu(r)$	*geometric quantity*
0	V	V Volume
1	$A/6$	A Surface
2	$H/(3\pi)$	H Mean curvature
3	χ	χ Euler characteristic

In collaboration with M. Kerscher (Munich University, Dept. Mathematics)

see also: Mecke, K.R., Buchert, T. and Wagner, H. (1994). Robust morphological measures for large scale structure in the universe. *Astron. Astrophys.*, **288**, 697-704.

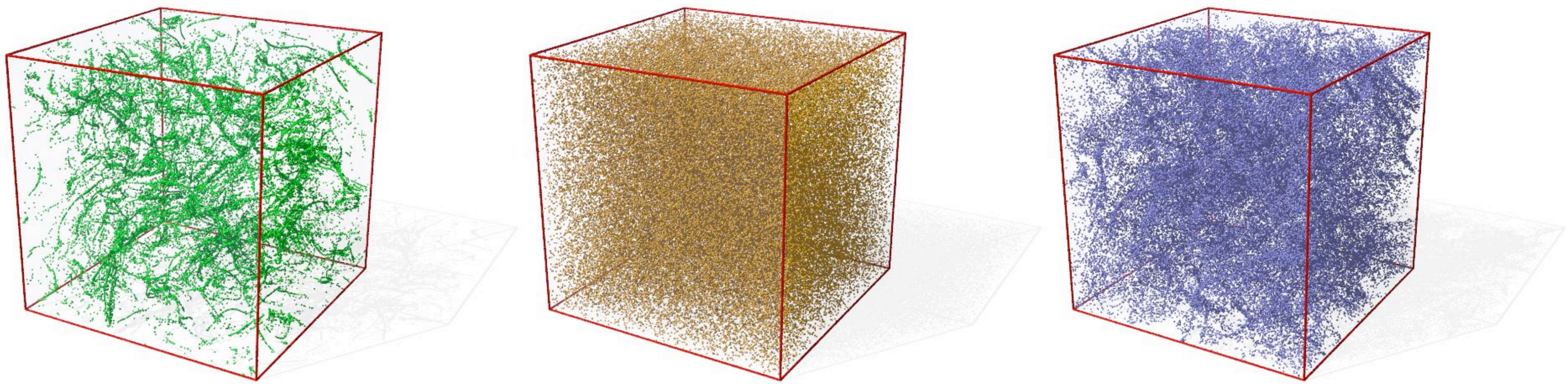
Comparison of three extreme cases

β = 0, heavy

β = 1, tracer — Poisson distribution

β = 3, bubble

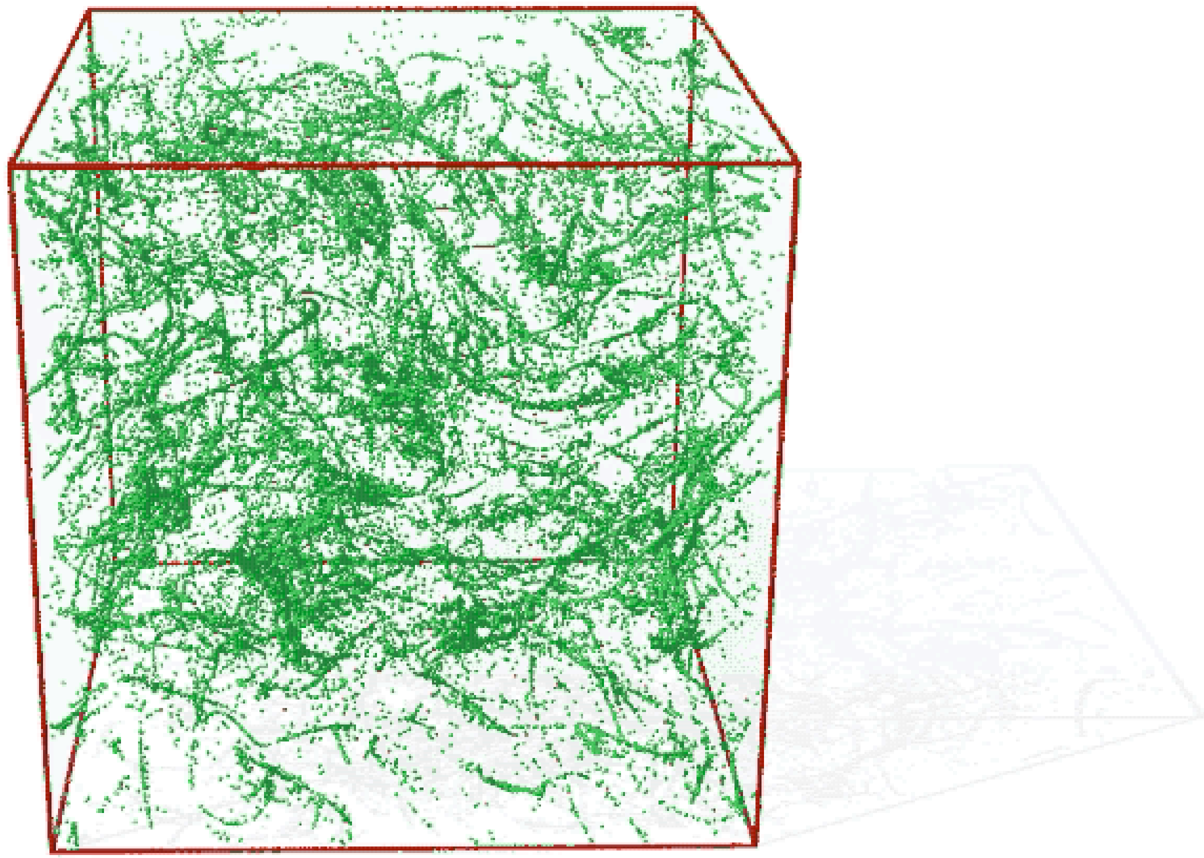
bubble tracer heavy



St=0.6 10^5 particles

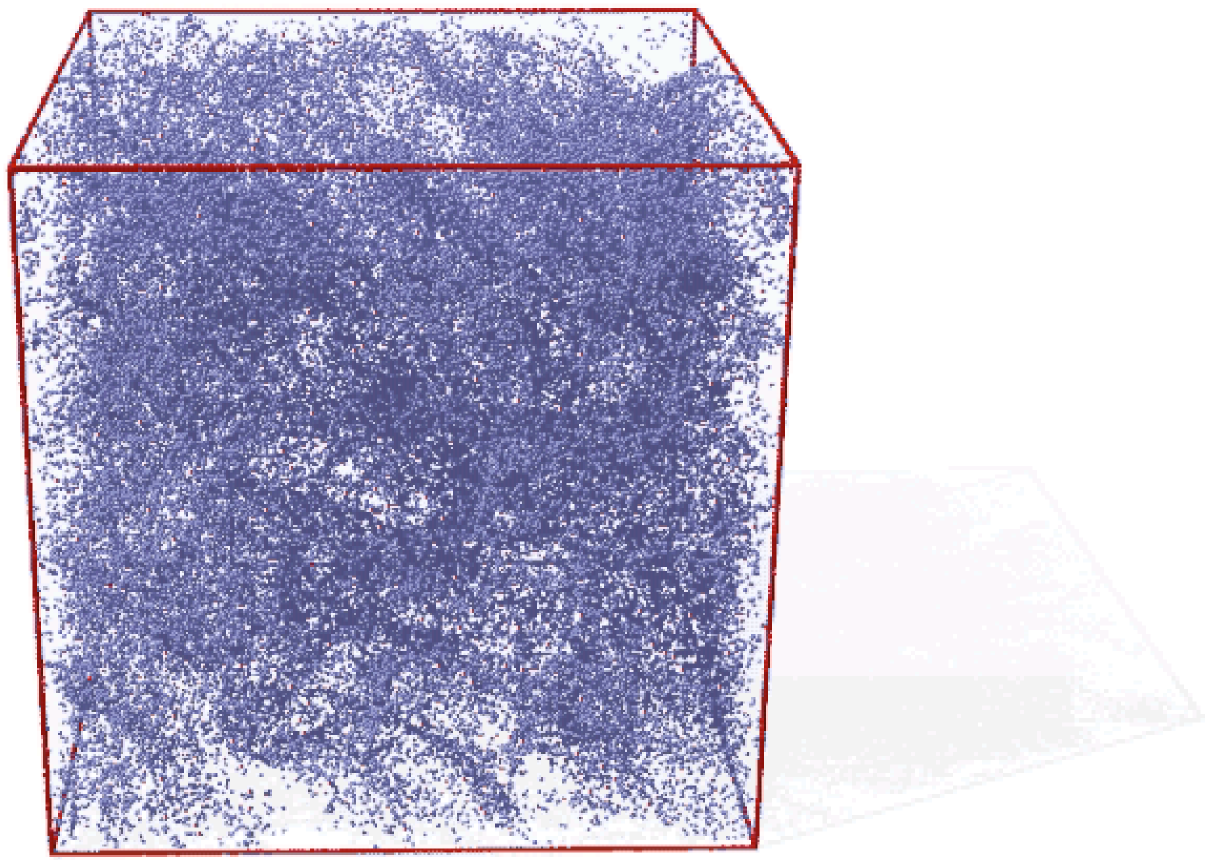
Bubbles β=3, St=0.6 and Re_λ=78

10^5
particles



Heavy particles β=0, St=0.6 and Re_λ=78

10^5
particles



Volume $V_0(r)$

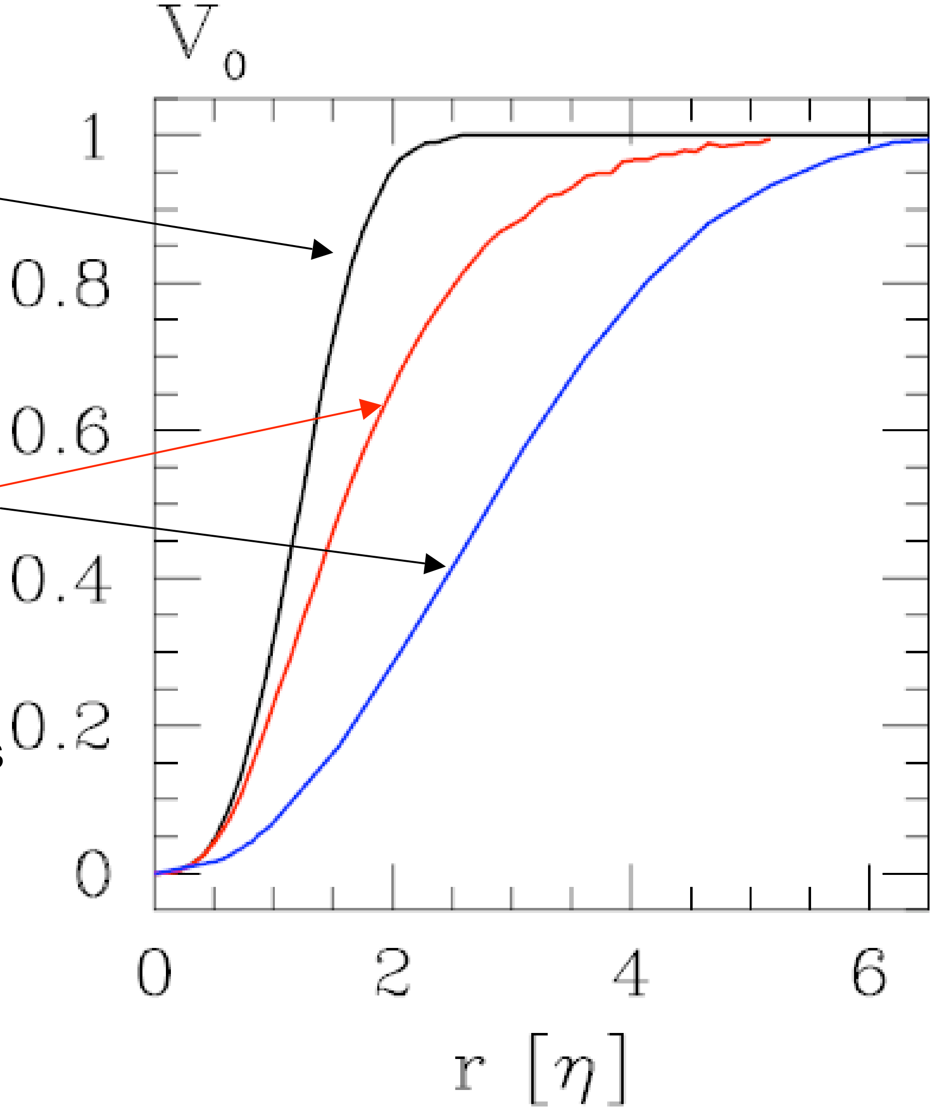
space-filling

space-filling

less fast due to

overlaps:

remote clusters



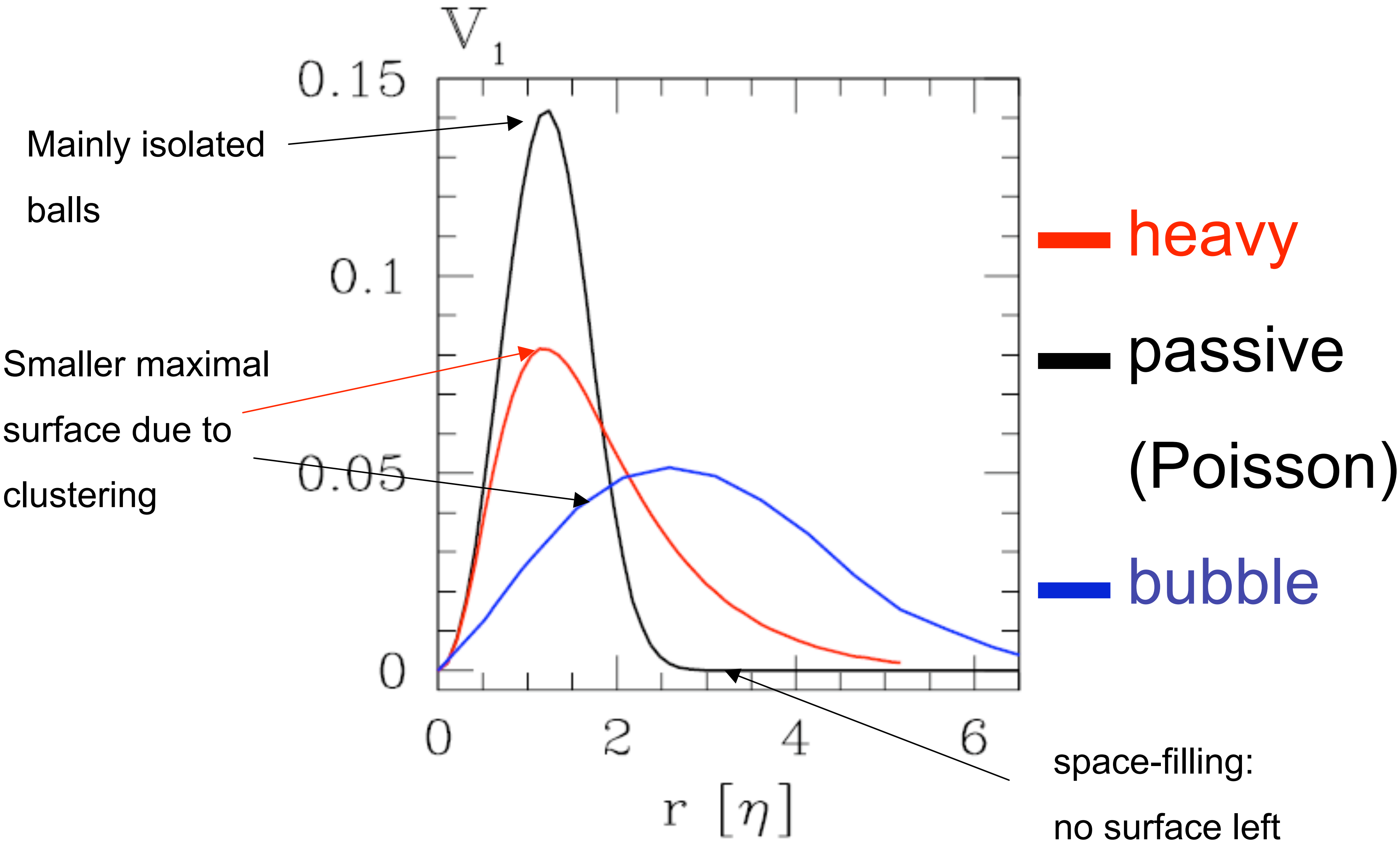
heavy

passive

(Poisson)

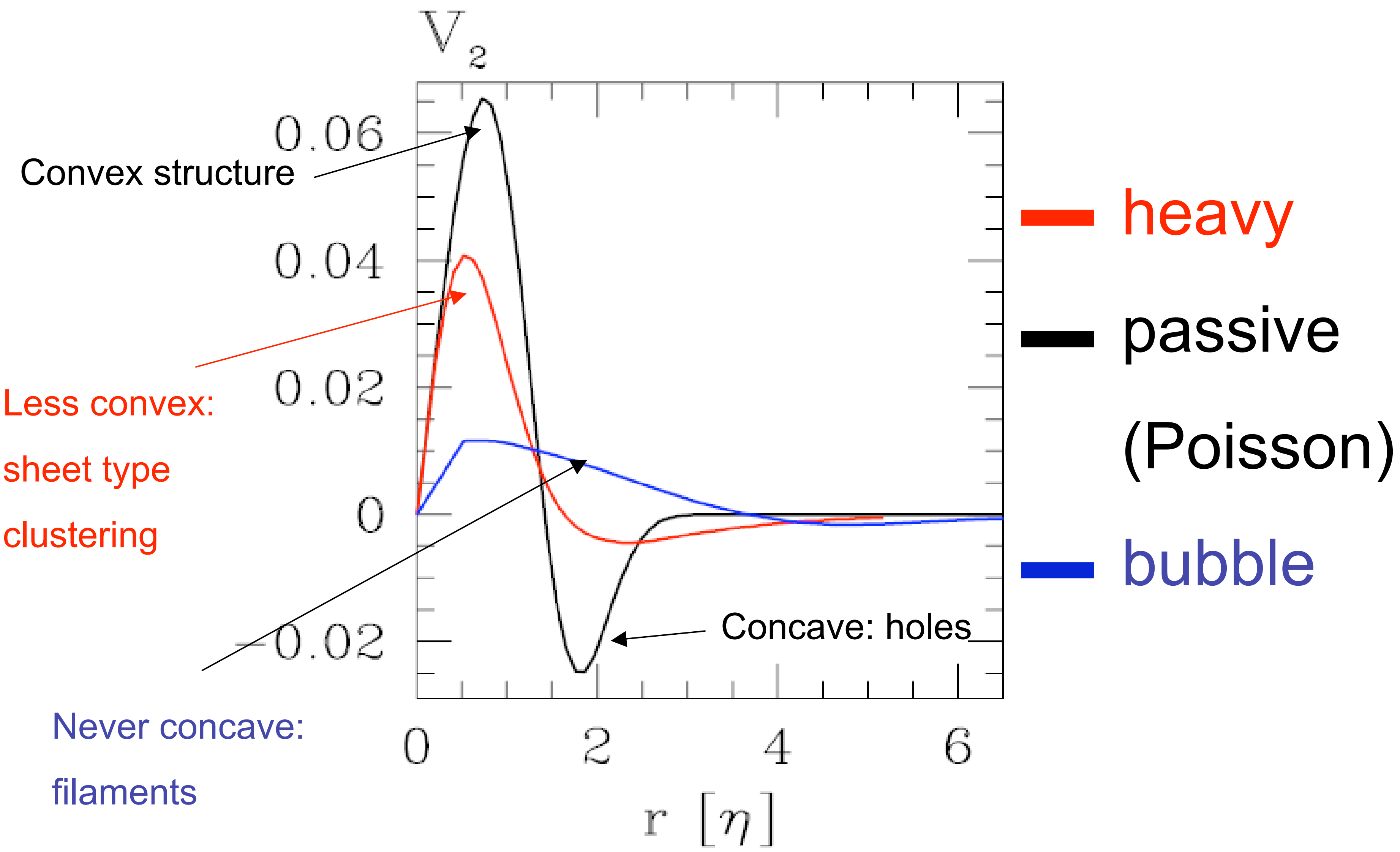
bubble

Surface $V_1(r)$



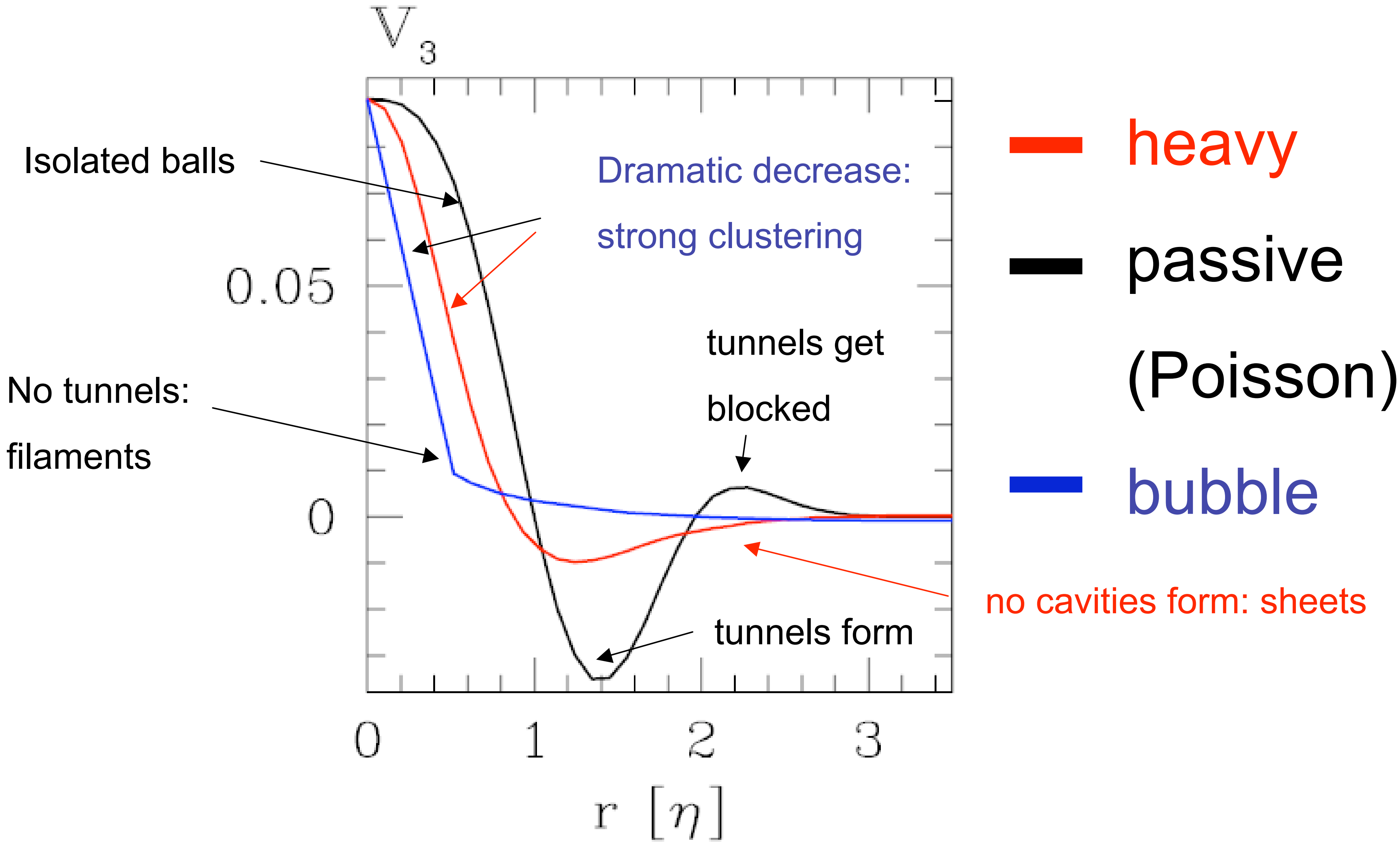
V1
0.15
0.1
0.05
0
0
2
4
6
r [η]
Mainly isolated balls
Smaller maximal surface due to clustering
heavy
passive (Poisson)
bubble
space-filling: no surface left

Mean curvature $V_2(r)$



Euler characteristics $V_3(r)$

$V_3 = \chi$ = n. components - n. tunnels + n.cavities



(ii) Concentration conditioned to local flow geometry

Eigenvalues of the strain matrix

$$\hat{\sigma}_{ij} = \partial_i u_j$$

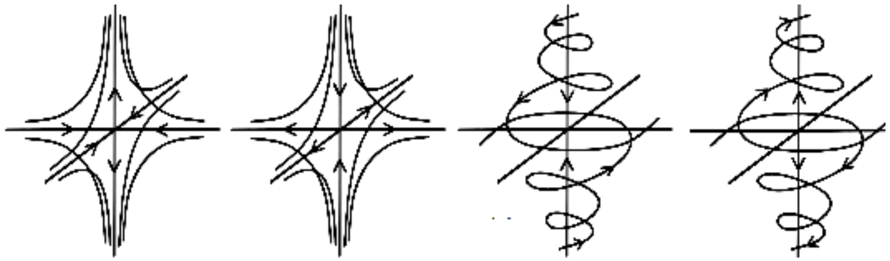
Discriminator:

$$\Delta = \left(\frac{\det[\hat{\sigma}]}{2}\right)^2 - \left(\frac{\mathrm{Tr}[\hat{\sigma}^2]}{6}\right)^3$$

$\boxed{\Delta \leq 0}$ 3 $\mathcal{R}$ eigen

$\Delta > 0$ 1 $\mathcal{R}$ + 2 $\mathcal{C}$ eigen.

Hyperbolic non-hyperbolic

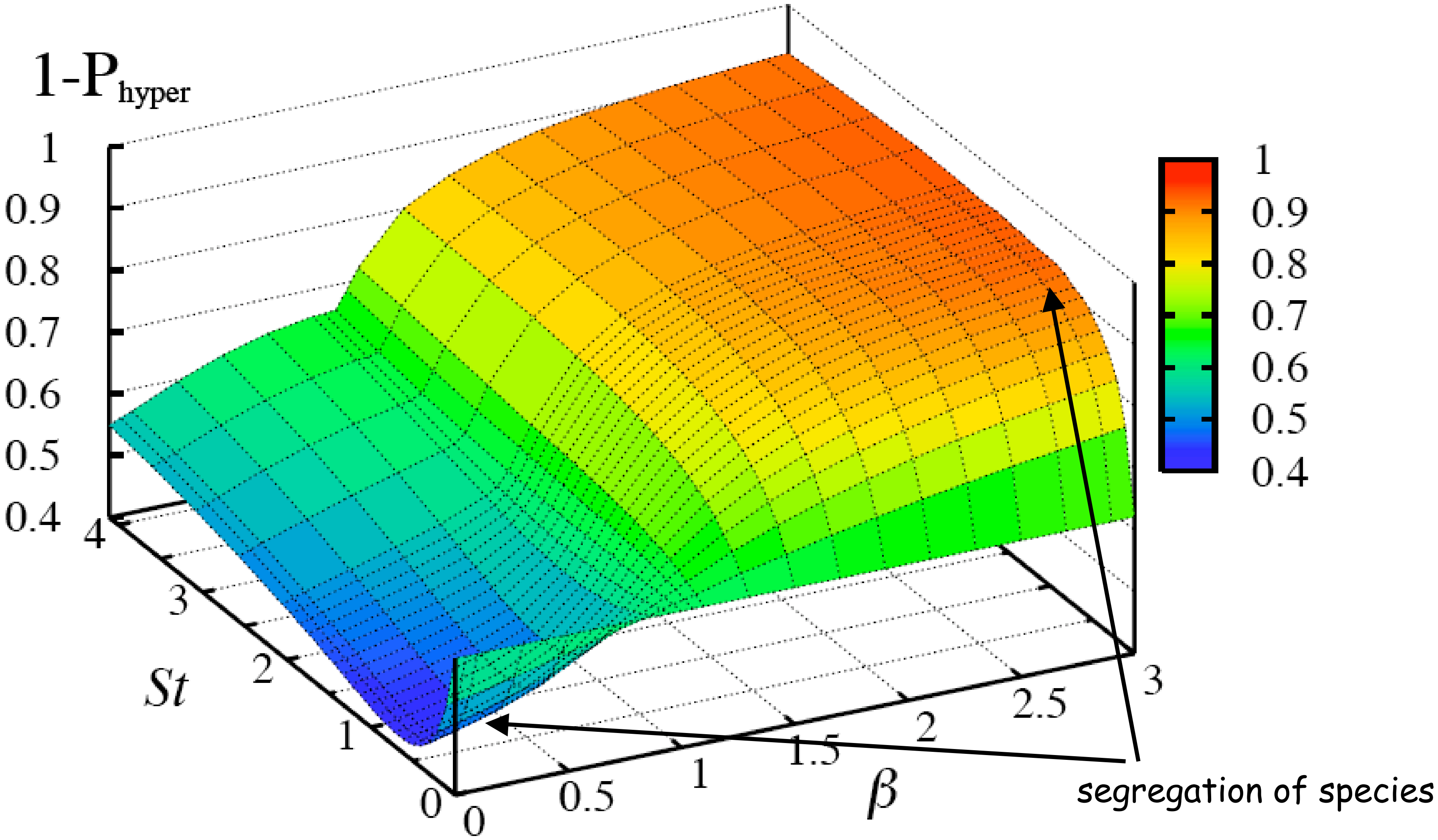


(see Chong, Perry, Cantwell PF 1990)

$$\boxed{P_{hyper} = \langle N(\Delta<0) / N_{tot} \rangle}$$

Conditioned concentration: 1-P_{hyper}

Probability to be in non-hyperbolic regions



Conclusions

Summary:

Small-scale clustering (dissipative range) quantified by:

D_{KY}, D_2 and Minkowski functionals.

Concentration conditioned to local flow geometry

(-> segregation) quantified by: P_{hyper}

Ongoing and future work:

- Quantifying clustering at larger scales (inertial-range).
- Investigate spatial statistics of dilute **bi-disperse** solutions.
- Trapping/ejection signature in temporal velocity statistics
- Trapping/ejection signature in acceleration statistics.
- Modeling needed.