SETTLING VELOCITY OF INERTIAL PARTICLES

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- effects of non-sphericity, rotationality and high velocity?

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  $\Longrightarrow$ possibility of variation $\mathrm{O}(\mathrm{St}^2)$ from bare value

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  $$\mathbf{u} = (\sin(kx_1)\cos[x_2 + \sin(\omega t)], -\cos(kx_1)\sin[x_2 + \sin(\omega t)]$$

  $\Longrightarrow$ dependence on oscillation and aspect ratio:

  $$\omega = 0, k = 1 \longleftrightarrow \begin{cases} \omega \neq 0, k = 1 \\ \end{cases}$$

CORRECTION AT ORDER St^2

- proportional to gravity ($\propto \mathrm{Fr}^{-2}$)
- same sign for both heavy and light particles ($\propto (1-\beta)^2$)
  $\Longrightarrow$ acceleration in falling and slowdown in rising, or viceversa
- actual presence and sign: dependent on the flow

Numerical examples:

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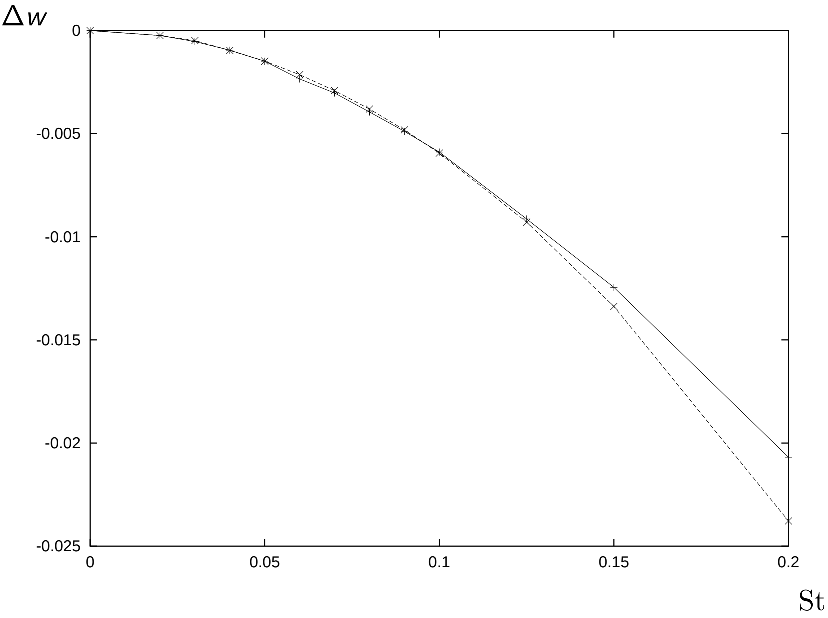
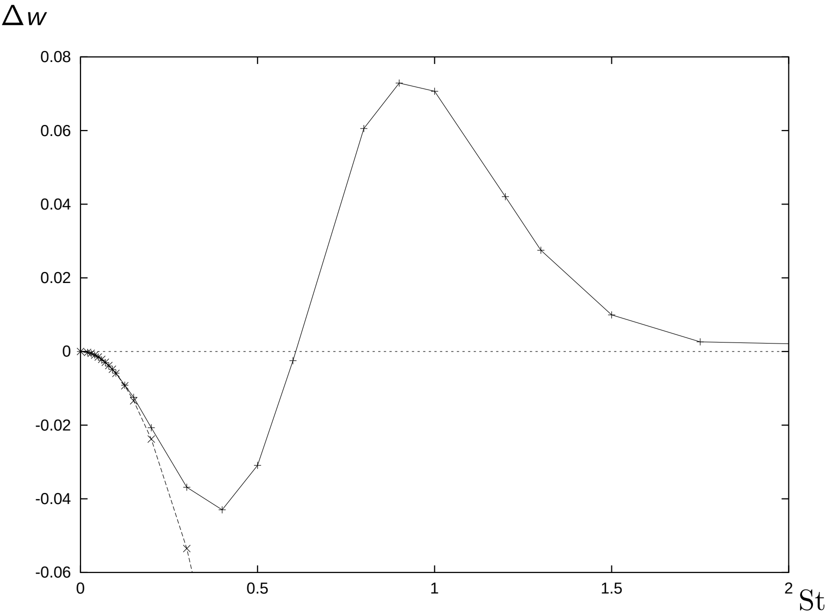

Δw
0
-0.005
-0.01
-0.015
-0.02
-0.025
0
0.05
0.1
0.15
0.2
St


Δw
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0
0.5
1
1.5
2
St

SINGULAR EXPANSIONS AND DIFFUSIVITY

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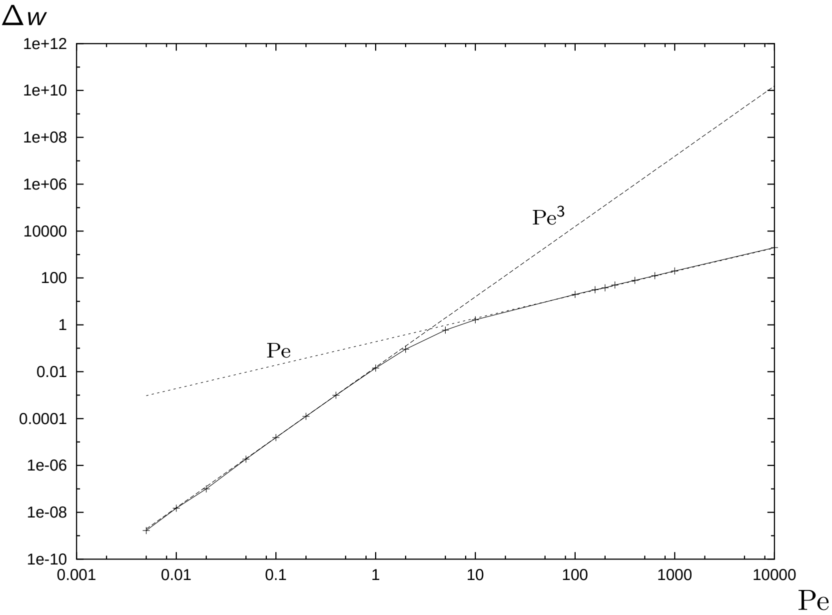
  - special case (stationary Gollub f.): behaviours Pe and Pe^3 respectively



Δw
1e+12
1e+10
1e+08
1e+06
10000
100
1
0.01
0.0001
1e-06
1e-08
1e-10
0.001
0.01
0.1
1
10
100
1000
10000
Pe
Pe³
Pe

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- (with P. Olla)
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- Study of effective diffusivity